V. *On the Theory of the Magneto-Optic Phenomena of Iron, Nickel, and Cobalt.*

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Introduction.

1. IN his 'British Association Report' (1893), on the "Action of Magnetism on Light," Mr. LARMOR points out that there are two possible ways in which the magnetic field may be regarded as affecting the phenomena of light propagation. "The imposed magnetisation is an independent kinetic system of a vortical character, which is linked on to the vibrational system which transmits the light waves," and from the first point of view "the kinetic reaction between the two systems will add on new terms to the electric force," and so there would be a "magneto-optic term" in the expression for the kinetic energy. This type of theory includes MAXWELL's hypothesis of molecular vortices, and has been analytically treated by FITZGERALD and BASSET; it also includes the theory developed by DRUDE in his paper "Magneto-optische Erscheinungen," in 'Wiedemann's Annalen,' vol. 46. The great difficulty arises when one comes to consider the boundary conditions, as a discontinuity of electric force cannot be avoided; apparently the only satisfactory way of meeting this difficulty is to be found in LARMOR's suggested modification of FITZGERALD's analysis, involving the supposition that in the case of reflection of light at the surface of a magnetised metal the constraint introduces an irrotational or compressional wave of the ether set up at the reflecting surface and travelling with very great or infinite velocity through the space occupied by the metal; a satisfactory system of equations of propagation and boundary conditions is thus obtained by applying the principle of Least Action. I have worked out the mathematics of this theory and obtained the general solution of the problem of reflection from a magnet; on comparing this with the experimental results of several German and Dutch physicists, it appears that the agreement of the theory with experiment is at best very doubtful, even when allowance is made for the possibility of large errors of observation. There is moreover one phenomenon, recently discovered, which the theory quite fails to account for, viz., an effect of the component of the magnetic field perpendicular to the plane of incidence.

2. The second type of theory supposes that the imposed magnetisation "slightly

alters the structure of the medium which conveys the light vibrations, but does not exert a direct dynamical effect on these vibrations"; the isotropy of the medium is, as it were, destroyed, and rotational terms appear in the fundamental elastic relations between displacements and the corresponding forces. This theory, in its valid form as regards boundary conditions, has quite lately been formulated for transparent media by BASSET ('American Journal of Mathematics,' vol. 19, 1897, No. 1), and the same principles underlie the very different analysis of GOLDHAMMER in his memoir of 1892 ('Wied. Ann.,' vol. 46). LARMOR also independently formulated this theory in his 'British Association Report,' 1893, and more explicitly in 'Proc. Lond. Math. Soc.,' April, 1893. In his exposition it was shown that the rotational terms in the equations connecting electric displacement and electric force are not open to the objection that they would imply perpetual motions, as they involve only the rate of change of the force. The boundary conditions in this theory are of the standard form, namely continuity of the tangential components of electric and magnetic force, and of the normal components of magnetic induction and total current. It has been shown by BASSET how the whole scheme may be formulated from a single energy function by the principle of Least Action.

3. In the present paper it is proposed to take the fundamental equations of this type of theory in a general form on the lines of Mr. LARMOR's recent papers on Electrodynamics, and to develop them so as to obtain the solutions of the problems of the reflection of light at the surface of a magnet, and of the transmission of light through normally magnetised metallic films. The formulæ so obtained will be compared with the available experimental results, with a view to ascertaining to what extent the theory is in agreement with the facts. The theory involves a single magneto-optic constant which in metals may be assumed complex; we shall try whether it is possible, by giving suitable numerical values to the modulus and vector angle of this constant, to make the theory account for all the observed phenomena; and if so, we shall ascertain what these numerical values are. If successful we shall thus have a formulation of the phenomena in a mathematical scheme, which ought to serve as a guide in the elaboration of physical theory.

In carrying out this programme, I am aware that I shall be going over ground which has already been covered to some extent by GOLDHAMMER, and also by DRUDE, but my method will be entirely different from theirs, and I shall be able to use important experimental results which had not been published at the time their papers were written.

Notation.

4. The notation is nearly the same as MAXWELL's: (P, Q, R) is electromotive force, (u, v, w) the total current, (a, b, c) magnetic induction, σ specific conductivity, K specific inductive capacity taken as a pure ratio, C the velocity of radiation; (f'', g'', h'') corresponds to MAXWELL's total electric displacement; its components

(f, g, h) and (f', g', h') are the vectors $\mathfrak{D}$, $\mathfrak{D}'$ of LARMOR's theory ('Phil. Trans.,' 1895), namely (f, g, h) is the displacement involved in the æther strain, and $(f', g' h')$ that involved in the polarisation of the matter.

Fundamental Equations.

5. It being as usual assumed that for oscillations so rapid as those of light the effective magnetic permeability is unity, the fundamental equations of the theory are as follows :—

(i.) The two circuital relations

$$\frac{dc}{dy} - \frac{db}{dz} = 4\pi u, \qquad \frac{da}{dz} - \frac{dc}{dx} = 4\pi v, \qquad \frac{db}{dx} - \frac{da}{dy} = 4\pi w \quad . \quad . \quad (1),$$

$$\frac{dR}{dy} - \frac{dQ}{dz} = -\frac{da}{dt}, \qquad \frac{dP}{dz} - \frac{dR}{dx} = -\frac{db}{dt}, \qquad \frac{dQ}{dx} - \frac{dP}{dy} = -\frac{dc}{dt} \quad . \quad . \quad (2).$$

(ii.) The equations of the current

$$\left.\begin{aligned} u &= \sigma P + g_3 Q - g_2 R + \frac{df''}{dt} \\ v &= \sigma Q + g_1 R - g_3 P + \frac{dg''}{dt} \\ w &= \sigma R + g_2 P - g_1 Q + \frac{dh''}{dt} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where the vector (g_1, g_2, g_3) represents the Hall effect.

(iii.) The displacement relations, and the elastic relations between electromotive force and the corresponding polarisation, viz.,

$$f'' = f + f', \quad g'' = g + g', \quad h'' = h + h'. \quad . \quad . \quad . \quad . \quad . \quad (4),$$

$$f = \frac{1}{4\pi C^2} P, \quad g = \frac{1}{4\pi C^2} Q, \quad h = \frac{1}{4\pi C^2} R. \quad . \quad . \quad . \quad . \quad . \quad (5),$$

and

$$\left.\begin{aligned} f' &= \frac{K-1}{4\pi C^2} P + b_3 \frac{dQ}{dt} - b_2 \frac{dR}{dt} \\ g' &= \frac{K-1}{4\pi C^2} Q + b_1 \frac{dR}{dt} - b_3 \frac{dP}{dt} \\ h' &= \frac{K-1}{4\pi C^2} R + b_2 \frac{dP}{dt} - b_1 \frac{dQ}{dt} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6),$$

the vector (b_1, b_2, b_3) representing, in transparent matter, the whole magneto-optic effect.

The restriction of the relation to this form is justified as follows :—

When there is no matter present the polarisation (f', g', h') is null, and there is no rotatory effect. When there is matter present the action of an electric field (P, Q, R) on the polarisation induced by it gives rise to a pondero-motive force which does mechanical work in a small displacement of the matter, equal, for an element of volume $\delta\tau$, to

$$(\mathrm{P}\,\delta f' + \mathrm{Q}\,\delta g' + \mathrm{R}\,\delta h')\,\delta\tau.$$

When there is no conduction and therefore no dissipation, this quantity must be an exact differential, otherwise mechanical work could be gained in a complete cycle of displacement of the material medium, which would imply the possibility of perpetual motions. This restriction must also hold universally, because the nature of the molecular polarisation is independent of whether conduction is present or not. It requires that $\int(\mathrm{P}\,\delta f' + \mathrm{Q}\,\delta g' + \mathrm{R}\,\delta h')\,\delta\tau$ and therefore $\int(f'\,\delta\mathrm{P} + g'\,\delta\mathrm{Q} + h'\,\delta\mathrm{R})\,d\tau$ shall be the variation of $\int \mathrm{F}\,d\tau$, where F is some function of (P, Q, R) and its differential coefficients, terms at the time limits being left out of account. The expression for (f', g', h') in terms of (P, Q, R) can therefore involve no rotatory terms in (P, Q, R) itself, as Lord KELVIN first shewed, but it may have rotatory terms in d/dt (P, Q, R), which have the characteristics of the magneto-optic property. Rotatory terms of a certain type in the spacial fluxions of (P, Q, R) are also admissible; these lead to optical rotation of the structural kind; they are foreign to the present problem because they are isotropic, instead of being related to an imposed vector, the intensity of magnetisation. In either case higher differentiations of odd order might also come into the expressions: these would affect the relations of the phenomena to optical dispersion, but not the questions here treated.

Equations of Propagation.

6 From the fundamental equations we readily obtain

$$\left.\begin{aligned}
u &= \left(\sigma + \frac{\mathrm{K}}{4\pi\mathrm{C}^2}\frac{d}{dt}\right)\mathrm{P} + \left(b_3\frac{d^2}{dt^2} + g_3\right)\mathrm{Q} - \left(b_2\frac{d^2}{dt^2} + g_2\right)\mathrm{R} \\
v &= \left(\sigma + \frac{\mathrm{K}}{4\pi\mathrm{C}^2}\frac{d}{dt}\right)\mathrm{Q} + \left(b_1\frac{d^2}{dt^2} + g_1\right)\mathrm{R} - \left(b_3\frac{d^2}{dt^2} + g_3\right)\mathrm{P} \\
w &= \left(\sigma + \frac{\mathrm{K}}{4\pi\mathrm{C}^2}\frac{d}{dt}\right)\mathrm{R} + \left(b_2\frac{d^2}{dt^2} + g_2\right)\mathrm{P} - \left(b_1\frac{d^2}{dt^2} + g_1\right)\mathrm{Q}
\end{aligned}\right\} \quad . \quad . \quad . \quad (7).$$

For brevity we may put

$$\left\{\left(b_1\frac{d^2}{dt^2} + g_1\right), \left(b_2\frac{d^2}{dt^2} + g_2\right), \left(b_3\frac{d^2}{dt^2} + g_3\right)\right\} \equiv \left\{\eta_1, \eta_2, \eta_3\right\} \quad . \quad . \quad . \quad (8)$$

and

$$\sigma + \frac{K}{4\pi C^2}\frac{d}{dt} \equiv \frac{1}{H} \quad \ldots \ldots \quad (9),$$

so that equations (7) become

$$\left.\begin{aligned} u &= \frac{1}{H}P + \eta_3 Q - \eta_2 R \\ v &= \frac{1}{H}Q + \eta_1 R - \eta_3 P \\ w &= \frac{1}{H}R + \eta_2 P - \eta_1 Q \end{aligned}\right\} \quad \ldots \ldots \quad (10).$$

Now (g_1, g_2, g_3) and (b_1, b_2, b_3) are supposed to be exceedingly small quantities, so that (η_1, η_2, η_3) are also extremely small. If we neglect squares and products of (η_1, η_2, η_3) and solve equations (10) for P, Q, R, we get

$$\left.\begin{aligned} P &= H(u - H\eta_3 v + H\eta_2 w) \\ Q &= H(v - H\eta_1 w + H\eta_3 u) \\ R &= H(w - H\eta_2 u + H\eta_1 v) \end{aligned}\right\} \quad \ldots \ldots \quad (11).$$

To get the equations of propagation differentiate with respect to the time the first of equations (1)

$$\begin{aligned} 4\pi\frac{du}{dt} &= \frac{d}{dz}\left(-\frac{db}{dt}\right) - \frac{d}{dy}\left(-\frac{dc}{dt}\right) \\ &= \frac{d}{dz}\left(\frac{dP}{dz} - \frac{dR}{dx}\right) - \frac{d}{dy}\left(\frac{dQ}{dx} - \frac{dP}{dy}\right) \text{ by (2)} \\ &= \nabla^2 P - \frac{d}{dx}\left(\frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz}\right) \\ &= H\nabla^2(u - H\eta_3 v + H\eta_2 w) \\ &\quad - H^2\frac{d}{dx}\left\{\eta_1\left(\frac{dv}{dz} - \frac{dw}{dy}\right) + \eta_2\left(\frac{dw}{dx} - \frac{du}{dz}\right) + \eta_3\left(\frac{du}{dy} - \frac{dv}{dx}\right)\right\}, \end{aligned}$$

and hence, if for brevity we put

$$\Omega' \equiv \eta_1\left(\frac{dw}{dy} - \frac{dv}{dz}\right) + \eta_2\left(\frac{du}{dz} - \frac{dw}{dx}\right) + \eta_3\left(\frac{dv}{dx} - \frac{du}{dy}\right) \quad \ldots \quad (12),$$

our equations of propagation become

$$\left.\begin{aligned} 4\pi\frac{du}{dt} &= H\nabla^2(u - H\eta_3 v + H\eta_2 w) + H^2\frac{d\Omega'}{dx} \\ 4\pi\frac{dv}{dt} &= H\nabla^2(v - H\eta_1 w + H\eta_3 u) + H^2\frac{d\Omega'}{dy} \\ 4\pi\frac{dw}{dt} &= H\nabla^2(w - H\eta_2 u + H\eta_1 v) + H^2\frac{d\Omega'}{dz} \end{aligned}\right\} \quad \ldots \quad (13).$$

Plane Waves in a Metallic Medium.

7. In the case of plane waves in a metallic medium, let us assume

$$(u,\ v,\ w) = (\mathrm{A},\ \mathrm{B},\ \mathrm{C})\, e^{\iota(lx+mz+pt)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where ι represents $\sqrt{-1}$, and write

$$l^2 + m^2 \equiv \omega^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15).$$

Substituting these values in the equations of propagation, we get

$$\left.\begin{aligned}
(\mathrm{H}\omega^2 + 4\pi\iota p)\,\mathrm{A} &= \mathrm{H}^2\omega^2(\eta_3\mathrm{B} - \eta_2\mathrm{C}) \\
&\qquad - \mathrm{H}^2 l\left\{\eta_1(-m\mathrm{B}) + \eta_2(m\mathrm{A} - l\mathrm{C}) + \eta_3(l\mathrm{B})\right\} \\
(\mathrm{H}\omega^2 + 4\pi\iota p)\,\mathrm{B} &= \mathrm{H}^2\omega^2(\eta_1\mathrm{C} - \eta_3\mathrm{A}) \\
(\mathrm{H}\omega^2 + 4\pi\iota p)\,\mathrm{C} &= \mathrm{H}^2\omega^2(\eta_2\mathrm{A} - \eta_1\mathrm{B}) \\
&\qquad - \mathrm{H}^2 m\left\{\eta_1(-m\mathrm{B}) + \eta_2(m\mathrm{A} - l\mathrm{C}) + \eta_3(l\mathrm{B})\right\}
\end{aligned}\right\} \quad . \quad (16).$$

Addition of l times the first of these to m times the last gives, as was to be expected,

$$l\mathrm{A} + m\mathrm{C} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17),$$

and hence if we eliminate A, B, and C, we get

$$\begin{vmatrix}
l & o & m \\
\mathrm{H}^2\omega^2\eta_3 & \mathrm{H}\omega^2 + 4\pi\iota p & -\mathrm{H}^2\omega^2\eta_1 \\
\mathrm{H}\omega^2 + 4\pi\iota p + \mathrm{H}^2 lm\eta_2 & -\mathrm{H}^2 lm\eta_1 - \mathrm{H}^2 m^2\eta_3 & \mathrm{H}^2 m^2\eta_2
\end{vmatrix} = 0,$$

which reduces to

$$(\mathrm{H}\omega^2 + 4\pi\iota p)^2 + \mathrm{H}^4\omega^2(l\eta_1 + m\eta_3)^2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (18).$$

This equation gives the possible values of m corresponding to given values of l and p. It is a quartic and therefore has four roots, of which two have their imaginary parts negative and their real parts positive; let us denote these roots by m_1 and m_2, and the corresponding values of ω by ω_1 and ω_2 respectively, so that

$$\left.\begin{aligned}
\mathrm{H}\omega_1^2 + 4\pi\iota p &= +\,\iota\,.\,\mathrm{H}^2\omega_1(l\eta_1 + m_1\eta_3) \\
\mathrm{H}\omega_2^2 + 4\pi\iota p &= -\,\iota\,.\,\mathrm{H}^2\omega_2^2(l\eta_1 + m_2\eta_3)
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19).$$

In the particular case of η_1, η_3, both zero, the equation to determine m would be

$$(\mathrm{H}\omega^2 + 4\pi\iota p)^2 = 0,$$

so that if M be the value of m given by this equation, and Ω the corresponding value of ω

and therefore

and

$$\left.\begin{aligned} \mathrm{H}\Omega^2 + 4\pi\iota p &= 0, \\ \Omega^2 &= -\frac{4\pi\iota p}{\mathrm{H}} \\ \mathrm{M}^2 &= -\frac{4\pi\iota p}{\mathrm{H}} - l^2 \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (20)$$

the values coinciding in pairs.

The sign of M is ambiguous; we determine it by requiring that M shall have its imaginary part negative, in which case we shall find that its real part is positive. As we neglect second and higher powers of η_1 and η_3, the equations (19) of the general case may now be written, introducing this quantity M,

$$\left.\begin{aligned} \omega_1^2 - \Omega^2 &= +\iota\,.\,\mathrm{H}\Omega\,(l\eta_1 + \mathrm{M}\eta_3) \\ \omega_2^2 - \Omega^2 &= -\iota\,.\,\mathrm{H}\Omega\,(l\eta_1 + \mathrm{M}\eta_3) \end{aligned}\right\} \quad \ldots \ldots \quad (21),$$

so that

$$\left.\begin{aligned} \omega_1^2 &= \Omega^2\left\{1 + \iota\,.\,\frac{\mathrm{H}}{\Omega}(l\eta_1 + \mathrm{M}\eta_3)\right\} \\ \omega_2^2 &= \Omega^2\left\{1 - \iota\,.\,\frac{\mathrm{H}}{\Omega}(l\eta_1 + \mathrm{M}\eta_3)\right\} \end{aligned}\right\} \quad \ldots \ldots \quad (22),$$

$$\left.\begin{aligned} \omega_1 &= \Omega\left\{1 + \iota\,.\,\frac{\mathrm{H}}{2\Omega}(l\eta_1 + \mathrm{M}\eta_3)\right\} \\ \omega_2 &= \Omega\left\{1 - \iota\,.\,\frac{\mathrm{H}}{2\Omega}(l\eta_1 + \mathrm{M}\eta_3)\right\} \end{aligned}\right\} \quad \ldots \ldots \quad (23),$$

$$\left.\begin{aligned} m_1^2 &= \mathrm{M}^2\left\{1 + \iota\,.\,\frac{\mathrm{H}\Omega}{\mathrm{M}^2}(l\eta_1 + \mathrm{M}\eta_3)\right\} \\ m_2^2 &= \mathrm{M}^2\left\{1 - \iota\,.\,\frac{\mathrm{H}\Omega}{\mathrm{M}^2}(l\eta_1 + \mathrm{M}\eta_3)\right\} \end{aligned}\right\} \quad \ldots \ldots \quad (24),$$

$$\left.\begin{aligned} m_1 &= \mathrm{M}\left\{1 + \iota\,.\,\frac{\mathrm{H}\Omega}{2\mathrm{M}^2}(l\eta_1 + \mathrm{M}\eta_3)\right\} \\ m_2 &= \mathrm{M}\left\{1 - \iota\,.\,\frac{\mathrm{H}\Omega}{2\mathrm{M}^2}(l\eta_1 + \mathrm{M}\eta_3)\right\} \end{aligned}\right\} \quad \ldots \ldots \quad (25),$$

results which will be of use later on.

Corresponding to the two values of m there are two sets of constants (A, B, C); these we distinguish by the suffixes ($_1$) and ($_2$).

Equation (17) shews that

$$C_1 = -\frac{l}{m_1} A_1 \quad \text{and} \quad C_2 = -\frac{l}{m_2} A_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (26).$$

This taken in conjunction with the second of equations (16) gives

$$(H\omega^2 + 4\pi\iota p) B = -\frac{H^2\omega^2}{m}(l\eta_1 + m\eta_3) A$$

whether the suffix be (1) or (2).

Hence in virtue of equations (19)

$$\iota \, . \, H^2\omega_1 B_1 = -\frac{H^2\omega_1^2}{m_1} A_1, \qquad -\iota \, . \, H^2\omega_2 B_2 = -\frac{H^2\omega_2^2}{m_2} A_2$$

or

$$B_1 = +\iota \, . \, \frac{\omega_1}{m_1} A_1, \qquad B_2 = -\iota \, . \, \frac{\omega_2}{m_2} A_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (27).$$

8. In the case of air (or any medium in which there is no magneto-optic rotation) η_1, η_2, η_3 are zero. For air also $\sigma = 0$, $K = 1$, $H = 4\pi c^2/\iota p = -\iota \, . \, 4\pi c^2/p$. The substitution of the exponential forms of u, v, w in the equations of propagation gives

$$-\iota \, . \, 4\pi c^2/p \, . \, \omega^2 + 4\pi\iota p = 0,$$

or $c^2\omega^2 = p^2$, as was to be expected.

Problem of Reflection.

9. We are now in a position to attack the problem of the reflection of light at the surface of a magnetised metal. Let the interface between the two media be the plane $z = 0$; the air occupying the space z positive, and the metal the space z negative. The plane of incidence is taken as the plane $y = 0$.

We assume that, *in the air*,

$$\left.\begin{aligned} u &= A_0 e^{\iota(lx + mz + pt)} + A e^{\iota(lx - mz + pt)} \\ v &= B_0 e^{\iota(lx + mz + pt)} + B e^{\iota(lx - mz + pt)} \\ w &= -\frac{l}{m} A_0 e^{\iota(lx + mz + pt)} + \frac{l}{m} A e^{\iota(lx - mz + pt)} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (28),$$

where A_0, B_0 represent the incident wave, and A, B the reflected wave.

In the metal,

$$\left.\begin{aligned} u &= \mathrm{A}_1 e^{\iota(lx + m_1 z + pt)} + \mathrm{A}_2 e^{\iota(lx + m_2 z + pt)} \\ v &= + \iota . \frac{\omega_1}{m_1} \mathrm{A}_1 e^{\iota(lx + m_1 z + pt)} - \iota \frac{\omega_2}{m_2} \mathrm{A}_2 e^{\iota(lx + m_2 z + pt)} \\ w &= - \frac{l}{m_1} \mathrm{A}_1 e^{\iota(lx + m_1 z + pt)} - \frac{l}{m_2} \mathrm{A}_2 e^{\iota(lx + m_2 z + pt)} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (29).$$

In this assumption we take account of only two of the four possible waves in the metallic medium; the other two are omitted because they are waves which travel in a direction that makes an acute angle with the axis of z; and as in our present problem all waves in the metallic medium are originated at the plane $z = 0$, only those can actually occur whose direction of propagation makes an obtuse angle with the axis of z.

10. The surface conditions which have to be satisfied are the continuity of

$$\begin{matrix} \mathrm{P}, & \mathrm{Q}, & w \\ a, & b, & c \end{matrix}$$

across the interface; and, as usual, the continuity of Q involves that of c, while the continuity of b involves that of w.

Thus the conditions are four, namely continuity of

$$\left.\begin{aligned} &(1)\ w \\ &(2)\ \mathrm{P, which} = \mathrm{H}(u - \mathrm{H}\eta_3 v + \mathrm{H}\eta_2 w) \\ &(3)\ \mathrm{Q, which} = \mathrm{H}(v - \mathrm{H}\eta_1 w + \mathrm{H}\eta_3 u) \\ &(4)\ a, \text{ which leads to the continuity of } d\mathrm{Q}/dz \text{ or } d/dz . \mathrm{H}(v - \mathrm{H}\eta_1 w + \mathrm{H}\eta_3 u) \end{aligned}\right\} \ (30).$$

We shall denote the H of the metallic medium by H′ to distinguish it from that of air. In the air $\mathrm{H} = -\iota . 4\pi \mathrm{C}^2/p$. If τ be the periodic time, and λ the wave-length in air of the light considered,

$$p = 2\pi/\tau = 2\pi \mathrm{C}/\lambda,$$

so that

$$\mathrm{H} = -2\iota \mathrm{C}\lambda.$$

Also since τ, and therefore p, is the same for the metal as for the air, we see by (20) that

$$\mathrm{H}'\Omega^2 = -4\pi\iota p = \mathrm{H}\omega^2$$

and therefore

$$\mathrm{H}/\mathrm{H}' = \Omega^2/\omega^2 = \mathrm{R}^2 e^{2\iota\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31),$$

where $Re^{\iota\alpha}$ is the quasi-refractive index of the metal (J. J. THOMSON, 'Recent Researches,' p. 419).

11. Let us now substitute the assumed exponential expressions for (u, v, w) in the surface conditions; we thus get

$$\left.\begin{aligned}
&-\frac{l}{m}A_0+\frac{l}{m}A=-\frac{l}{m_1}A_1-\frac{l}{m_2}A_2\\
&H(A_0+A)=H'(A_1+A_2)\\
&\qquad -H'^2\left\{\eta_3\left(\iota\frac{\omega_1}{m_1}A_1-\iota\frac{\omega_2}{m_2}A_2\right)-\eta_2\left(-\frac{l}{m_1}A_1-\frac{l}{m_2}A_2\right)\right\}\\
&H(B_0+B)=H'\left(\iota\frac{\omega_1}{m_1}A_1-\iota\frac{\omega_2}{m_2}A_2\right)\\
&\qquad -H'^2\left\{\eta_1\left(-\frac{l}{m_1}A_1-\frac{l}{m_2}A_2\right)-\eta_3(A_1+A_2)\right\}\\
&H\iota(mB_0-mB)=H'(-\omega_1A_1+\omega_2A_2)\\
&\qquad -H'^2\left\{\eta_1(-\iota lA_1-\iota lA_2)-\eta_3(\iota m_1A_1+\iota m_2A_2)\right\}
\end{aligned}\right\} \quad (32).$$

We may make the form of the first of these equations analogous with that of the others by multiplying it across by $H\omega^2$ or $H'\Omega^2$. The four equations may then be written as follows:

$$\left.\begin{aligned}
&H(A_0+A)=H'(A_1+A_2)-H'^2\iota.\eta_3\left(\frac{\omega_1}{m_1}A_1-\frac{\omega_2}{m_2}A_2\right)-H'^2\eta_2 l\left(\frac{A_1}{m_1}+\frac{A_2}{m_2}\right)\\
&H\frac{\omega^2}{m}(A_0-A)=H'\Omega^2\left(\frac{A_1}{m_1}+\frac{A_2}{m_2}\right)\\
&-H\iota.(B_0+B)=H'\left(\frac{\omega_1}{m_1}A_1-\frac{\omega_2}{m_2}A_2\right)-H'^2\iota\eta_3(A_1+A_2)-H'^2\iota\eta_1 l\left(\frac{A_1}{m_1}+\frac{A_2}{m_2}\right)\\
&-Hm\iota(B_0-B)=H'(\omega_1A_1-\omega_2A_2)-H'^2\iota\eta_3(m_1A_1+m_2A_2)-H'^2\iota\eta_1 l(A_1+A_2)
\end{aligned}\right\} \quad (33).$$

When we substitute in these the values of ω_1, m_1, ω_2, m_2 found above, and neglect small quantities of the second and higher orders, we get

$$\left.\begin{aligned}
&H(A_0+A)=H'\left\{1-H'\eta_2\frac{l}{M}\right\}(A_1+A_2)-H'^2\iota\eta_3\frac{\Omega}{M}(A_1-A_2)\\
&H\frac{\omega^2}{m}(A_0-A)=H'\frac{\Omega^2}{M}(A_1+A_2)-\tfrac{1}{2}H'^2\iota.\frac{\Omega^3}{M^3}(l\eta_1+M\eta_3)(A_1-A_2)\\
&-H\iota(B_0+B)=H'\frac{\Omega}{M}(A_1-A_2)-\tfrac{1}{2}H'^2\iota\frac{(\Omega^2+M^2)}{M^3}(l\eta_1+M\eta_3)(A_1+A_2)\\
&-Hm\iota(B_0-B)=H'\Omega(A_1-A_2)-\tfrac{1}{2}H'^2\iota.(l\eta_1+M\eta_3)(A_1+A_2)
\end{aligned}\right\} \quad (34).$$

From the second and third of these equations we readily get

$$H' \frac{\Omega}{M} (A_1 - A_2) = - H\iota (B_0 + B) + \tfrac{1}{2} H' \iota \frac{(\Omega^2 + M^2)}{M^2 \Omega^2} (l\eta_1 + M\eta_3) H \frac{\omega^2}{m} (A_0 - A),$$

and

$$H' \frac{\Omega^2}{M} (A_1 + A_2) = H \frac{\omega^2}{m} (A_0 - A) + \tfrac{1}{2} H' \iota \frac{\Omega^2}{M^2} (l\eta_1 + M\eta_3) (- H\iota) (B_0 + B).$$

Substituting from these for $(A_1 - A_2)$ and $(A_1 + A_2)$ in the first and fourth of (34), remembering (31), and for brevity denoting $R^2 e^{2\iota\alpha}$ by μ, we get

$$\mu (A_0 + A) = \left(1 - H' \frac{l}{M} \eta_2\right) \frac{M}{m} (A_0 - A) + \tfrac{1}{2} \frac{\mu}{M} H' (l\eta_1 - M\eta_3) (B_0 + B),$$

$$m (B_0 - B) = M (B_0 + B) - \tfrac{1}{2} H' \frac{\omega^2}{Mm} (l\eta_1 + M\eta_3) (A_0 - A),$$

which may be written

$$\left.\begin{aligned} &\left\{\mu + \frac{M}{m}\left(1 - H' \frac{l}{M} \eta_2\right)\right\} A - \tfrac{1}{2} \frac{\mu H'}{M} (l\eta_1 - M\eta_3) B \\ &\qquad + \left\{\mu - \frac{M}{m}\left(1 - H' \frac{l}{M} \eta_2\right)\right\} A_0 - \tfrac{1}{2} \frac{\mu H'}{M} (l\eta_1 - M\eta_3) B_0 = 0 \\ &\tfrac{1}{2} H' \frac{\omega^2}{Mm} (l\eta_1 + M\eta_3) A + (m + M) B - \tfrac{1}{2} H' \frac{\omega^2}{Mm} (l\eta_1 + M\eta_3) A_0 - (m - M) B_0 = 0 \end{aligned}\right\} (35),$$

and solving these for A and B, we get

$$\left.\begin{aligned} &\frac{A}{- \left\{R^2 e^{2\iota\alpha} - \frac{M}{m}\left(1 + \iota \frac{2c\lambda}{R^2 e^{2\iota\alpha}} \frac{l}{M} \eta_2\right)\right\} (m + M) A_0 - 2\iota c\lambda \frac{m}{M} (l\eta_1 - M\eta_3) B_0} \\ &= \frac{B}{- 2\iota c\lambda \frac{\omega^2}{Mm} (l\eta_1 + M\eta_3) A_0 + \left\{R^2 e^{2\iota\alpha} + \frac{M}{m}\left(1 + \iota \frac{2c\lambda}{R^2 e^{2\iota\alpha}} \frac{l}{M} \eta_2\right)\right\} (m - M) B_0} \\ &= \frac{1}{\left\{R^2 e^{2\iota\alpha} + \frac{M}{m}\left(1 + \iota \frac{2c\lambda}{R^2 e^{2\iota\alpha}} \frac{l}{M} \eta_2\right)\right\} (m + M)} \end{aligned}\right\} (36),$$

which is the complete mathematical solution of the problem of reflection.

Relation of η_1, η_2, η_3 to the Magnetic Field.

12. Before going further, we must consider how (η_1, η_2, η_3) depend on the imposed magnetic field; and first it should be noticed that, though (η_1, η_2, η_3) appear as

operators, in the case of light vibrations they are algebraical quantities, because $d^2/dt^2 = -p^2$.

In theories of this nature it is usual to assume that (b_1, b_2, b_3) are proportional to the components, parallel to the axes, of the imposed magnetic force; the constants (g_1, g_2, g_3) of the HALL effect are also usually assumed to vary as the magnetic force; and therefore in this case (η_1, η_2, η_3) would do so also. But in experiments on the transmission of light through magnetised metallic films it is found that the rotation of the plane of polarisation certainly does not vary as the magnetic force, but very probably varies as the intensity of magnetisation, a quantity very difficult to determine. We shall see later that in the mathematical solution of the problem of transmission the rotation varies as η_3, and hence we are driven to the assumption that $(\eta_1\ \eta_2\ \eta_3)$ vary as the components of magnetisation. I shall also suppose that (b_1, b_2, b_3) vary as the components of magnetisation, which necessitates the assumption that (g_1, g_2, g_3) do so likewise; the question whether the HALL effect varies as the magnetic force or as the magnetisation has not, I think, been put to an experimental test; the latter supposition seems more probable.

These assumptions can be readily justified from physical considerations. For *in vacuo* there is no magneto-optic rotation, though there is magnetic force; it is therefore not the magnetic force, but matter, or some property of matter when under the influence of magnetic force, that causes the rotation; and the property of matter under the influence of magnetic force is not force, but magnetisation.

Denoting the components of the imposed magnetisation by $(\alpha_0, \beta_0, \gamma_0)$, we assume

$$(\eta_1, \eta_2, \eta_3) = C_0 e^{\iota x} (\alpha_0, \beta_0, \gamma_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37),$$

where $C_0 e^{\iota x}$ is the complex magneto-optic constant of the theory. For any particular metal the values of C_0 and x may be determined by experiment; and if we find that the numerical values of these constants, as determined by all the different sorts of experiments, are the same, we shall conclude that the theory can account for all the observed facts, and therefore constitutes a complete mathematical explanation of the phenomena.

The Optic Constants of Metals.

13. The constants R and α are different for different metals, and also for light of different colours. Their values have not been directly tabulated, but they are easily obtained from the tabulated values of DRUDE'S optic constants; these latter are denoted by n and k, and are connected with R and α by the relations

$$R \cos 2\alpha = n^2 (1 - k^2), \qquad R \sin 2\alpha = -2n^2 k,$$

so that

$$R^2 = n^2 (1 + k^2), \qquad \tan \alpha = -k.$$

The values of n and k for iron, steel, and nickel will be found in a paper of DRUDE's ('Wied. Ann.,' vol. 39, p. 481), quoted in THOMSON's 'Recent Researches,' p. 421. The constants for cobalt are given by DRUDE in 'Wied. Ann.,' vol. 46, p. 407. These values are shewn in the following table :—

	Red light.			Sodium light.		
	nk.	n.	k.	nk.	n.	k.
Iron	..	..	..	3·20	2·36	1·36
Steel	3·47	2·62	1·32	3·40	2·41	1·38
Nickel.	3·56	1·89	1·88	3·32	1·79	1·86
Cobalt	4·19	2·22	1·89	4·03	2·12	1·90

wherein, for red light, $\lambda = 630 \times 10^{-7}$ centim., and for sodium light we may take $\lambda = 589{\cdot}6 \times 10^{-7}$ centim.

Hence we find the corresponding values of R and α.

	Red light.		Sodium light.	
	R^2.	$-\alpha$.	R^2.	$-\alpha$.
Iron	..	..	15·86	53° 40′
Steel	18·82	52° 51′	16·87	54° 4′
Nickel.	16·20	62° 0′	14·29	61° 44′
Cobalt	22·48	62° 7′	20·72	62° 14′

The KERR *Experiments.*

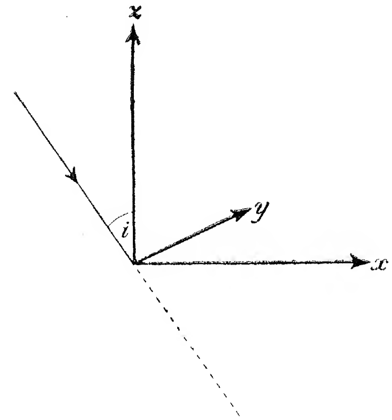
14. We shall first compare our theory with the results of the KERR experiments, which are so well known that they need not be here described.

In Dr. KERR's second experiment the magnetisation is parallel to the reflecting surface, and to the plane of incidence; and the incident light is polarised perpendicularly to the plane of incidence. Thus, in our notation, $\beta_0 = 0$, $\gamma_0 = 0$, and $B_0 = 0$; and the reflected light is specified by A and B, whose values as given by formula (36) are

$$\left.\begin{aligned} A &= \frac{-(R^2e^{2\iota\alpha} - M/m)(M+m)A_0}{(R^2e^{2\iota\alpha} + M/m)(M+m)} \\ B &= \frac{-2\iota c\lambda(\omega^2/Mm)l\eta_1 A_0}{(R^2e^{2\iota\alpha} + M/m)(M+m)} \end{aligned}\right\} \quad \ldots \ldots \ldots (38).$$

If the incident ray be as represented in the figure, and i be the angle of incidence, and p and ω be positive, then

$$l = -\omega \sin i$$
$$m = +\omega \cos i.$$



The incident ray being plane polarised, A_0 is real. But A and B are both complex, and have not necessarily the same vector angle; hence the reflected light is elliptically polarised. If θ be the angle through which the major axis of the ellipse of polarisation is rotated round the reflected ray (in the direction from the axis of x towards the axis of y) from the plane $x = 0$, since the modulus of B is very small compared with the modulus of A, θ is given by

$$\theta = \text{real part of } (\mathrm{B} \cos i/\mathrm{A})$$

in circular measure.

In the case of iron, KERR found that when α_0 is negative, θ is negative if i be less than about 75°; while if i be greater than 75°, θ is positive. The angle of incidence for which θ changes sign (and therefore vanishes) has been observed by different experimenters, whose results differ considerably. They are as follows:—

KERR 75°.
KUNDT 80° to 82°.
RIGHI 78° 54′.
SISSINGH 80°.
DRUDE 79°.

Now from (38)

$$\frac{\mathrm{B}}{\mathrm{A}} = \frac{2\iota c\lambda \dfrac{\omega^2}{\mathrm{M}m} l\mathrm{C}_0 e^{\iota x}\alpha_0}{(\mathrm{R}^2 e^{2\iota\alpha} - \mathrm{M}/m)(\mathrm{M} + m)}$$

so that

$$\frac{\mathrm{B}\cos i}{\mathrm{A}} = \frac{2c\lambda \sin i \cos i\, \mathrm{C}_0\alpha_0 . \iota . e^{\iota x}}{\mathrm{R}^2 e^{2\iota\alpha} . \mathfrak{M} . (\mathfrak{M} + \cos i)(\mathfrak{M}\mathrm{R}^{-2}e^{-2\iota\alpha} - \cos i)} \quad . \quad . \quad . \quad . \quad (38^*),$$

where

$$\mathfrak{M} = \mathrm{M}/\omega,$$

and, therefore, since

$$\omega^2 \mathrm{R}^2 e^{2\iota\alpha} = \Omega^2 = \mathrm{M}^2 + \omega^2 \sin^2 i$$

$$\mathfrak{M}^2 = \mathrm{R}^2 e^{2\iota\alpha} - \sin^2 i \quad . \quad . \quad . \quad . \quad (39),$$

θ changes sign for that value of i which makes the vector angle of $B \cos i/A$ equal to an odd number of right angles. Let us assume that x must lie between 0° and 180°, leaving the sign of C_0 to be determined afterwards. To obtain the vector angle for any given angle of incidence we must calculate the vector angles of the various complex factors which occur in numerator and denominator of the fraction in equation (38*). This involves troublesome arithmetical work ; but it is preferable to the approximation on the supposition that R is large, used by J. J. THOMSON ('Recent Researches,' p. 498) in a similar investigation, as that method introduces an error of quite a large number of degrees.

Using the constants for yellow light, I get the following values :—

Angle of incidence.	Vector angle of $\mathfrak{M}$.	Vector angle of $\mathfrak{M} + \cos i$.	Vector angle of $\mathfrak{M}R^{-2}e^{-2\iota\alpha} - \cos i$
75°	−55° 15′	−52° 20′	117° 17′
78° 54′	−55° 18′	−53° 6′	100° 23′
80°	−55° 19′	−53° 19′	95° 2′

whence are derived the following :—

	Angle of incidence.	Vector angle of $(B \cos i/A)$.
KERR	75°	x + 187° 38′
RIGHI	78° 54′	x + 205° 21′
SISSINGH	80°	x + 210° 56′

So that if θ changes sign when $i = 75°$,

$$x = 82° \; 22'.$$

If when $i = 78° \; 54'$, then

$$x = 64° \; 39'.$$

If when $i = 80°$, then

$$x = 59° \; 4'.$$

And probably if θ changed sign when $i = 78°$, the corresponding value of x would be about 69°.

The uncertainty as to the exact value of the angle of incidence for which θ vanishes, and the large difference, caused by a small error of observation, in the resulting value of x, render this experiment unsuitable as a means of arriving at the exact value of x. It will, however, be useful in testing a value of x determined in some other way.

The experiment will also tell us the sign of C_0; for, in accordance with KERR's observations, when the incidence is very nearly normal, θ is of the same sign as α_0; and when the angle of incidence is nearly 90°, θ is of the opposite sign to α_0. Now the table of values given above indicates that as i passes through that value (be it 75° or 80°) for which θ vanishes, from a less to a greater value, the cosine of the vector angle from being negative becomes positive, so that for very great angles of incidence θ is of the same sign as $C_0\alpha_0$. Hence C_0 is negative.

15. In KERR's first experiment the magnetisation is parallel to the reflecting surface and the incident light is polarised in the plane of incidence. If θ be the rotation of the major axis of the ellipse of polarisation in the same sense as before, KERR found that θ has the same sign for all angles of incidence, and that this sign is opposite to that of α_0.

In this case $\eta_2 = 0$, $\eta_3 = 0$, $A_0 = 0$, and $\theta =$ real part of $(-A/B \cos i)$.

From result (36) we readily deduce that

$$\frac{A}{B \cos i} = \frac{-2c\lambda \sin i \cos i \, C_0\alpha_0\iota \, . \, e^{\iota x}}{R^2 e^{2\iota\alpha} \mathfrak{M} (\mathfrak{M} - \cos i)(\mathfrak{M} R^{-2} e^{-2\iota\alpha} + \cos i)},$$

of which the vector angle (including the minus sign) is $x - 90° - 2\alpha -$ sum of vector angles of $\mathfrak{M}$, $(\mathfrak{M} - \cos i)$, $(\mathfrak{M} R^{-2} e^{-2\iota\alpha} + \cos i)$.

If $i = 0$, vector angle of $A/B \cos i$ is

$$x + 128° \, 3'.$$

If $i = 61° \, 30'$ vector angle is

$$x + 115° \, 42'.$$

If $i = 90°$, vector angle is

$$x + 76° \, 4'.$$

And evidently for all angles of incidence the vector angle lies between $x + 76°$ and $x + 128°$. So that if x have any value between 14° and 142°, the cosine of the vector angle of $A/B \cos i$ is negative for all angles of incidence. Thus θ has always the same sign as $C_0\alpha_0$, that is, the opposite sign to α_0.

Hence any value of x lying between 14° and 142° satisfies all the conditions of KERR's first experiment.

16. In another of KERR's experiments the magnetisation is normal to the reflecting surface, and the incident light is polarised in the plane of incidence.

Here

$$\eta_1 = 0, \qquad \eta_2 = 0, \qquad A_0 = 0;$$

θ, having the same meaning as before, is found by KERR to be of opposite sign to γ_0 for all angles of incidence. Now θ is the real part of $-A/B \cos i$, and we readily deduce from formula (36) that

$$\frac{\mathrm{A}}{\mathrm{B}\cos i} = \frac{2c\lambda \cos i \mathrm{C}_0\gamma_0 (-\iota)\, e^{\iota x}}{\mathrm{R}^2 e^{2\iota a} (\mathfrak{M} - \cos i)(\mathfrak{M}\mathrm{R}^{-2}e^{-2\iota a} + \cos i)},$$

of which complex the vector angle is found in the usual way to lie, for all angles of incidence, between the values $x + 20° \, 43'$, corresponding to $i = 90°$, and $x + 74° \, 23'$, corresponding to $i = 0$.

If x have any value between $69° \, 17'$ and $195° \, 37'$, the cosine of this vector angle is always negative; and so θ has the same sign as $\mathrm{C}_0\gamma_0$, or the opposite sign to γ_0. Hence this experiment of KERR's is satisfied if x have any value between $69° \, 17'$ and $195° \, 37'$.

17. In KERR's fourth experiment the magnetisation is normal to the reflecting surface, and the incident light is polarised perpendicularly to the plane of incidence. Here $\eta_1 = 0$, $\eta_2 = 0$, $\mathrm{B}_0 = 0$, and θ is the real part of $\mathrm{B}\cos i/\mathrm{A}$ where, from (36),

$$\frac{\mathrm{B}\cos i}{\mathrm{A}} = \frac{2c\lambda \cos i \mathrm{C}_0\gamma_0 (-\iota)\, e^{\iota x}}{\mathrm{R}^2 e^{2\iota a} (\mathfrak{M} + \cos i)(\mathfrak{M}\mathrm{R}^{-2}e^{-2\iota a} - \cos i)},$$

of which complex the vector angle lies between the values $x - 105° \, 37'$, corresponding to $i = 0$, and $x + 20° \, 43'$, corresponding to $i = 90°$.

Now KERR found that θ is, for all angles of incidence, of opposite sign to γ_0, that is, of the same sign as $\mathrm{C}_0\gamma_0$. Hence the cosine of the vector angle of $\mathrm{B}\cos i/\mathrm{A}$ is always positive. This is in accordance with our theory, provided the value of x lie between $15° \, 37'$ and $69° \, 17'$.

Obviously this conclusion is at variance with that derived from the preceding experiment, unless x happen to have exactly the value $69° \, 17'$.

But this experiment of KERR's was repeated by KUNDT, who found that θ has not the same sign for all angles of incidence, but that it vanishes and changes sign for an angle of incidence which he estimated at about 82°.

I have calculated the values of the vector angle of $\mathrm{B}\cos i/\mathrm{A}$ for several angles of incidence in the neighbourhood of 82°; they are as follows:—

Angle of incidence.	Vector angle of $\mathrm{B}\cos i/\mathrm{A}$.	Angle of incidence.	Vector angle of $\mathrm{B}\cos i/\mathrm{A}$.
75°	$x - 47° \, 37'$	85°	$x + 0° \, 43'$
78° 54′	$x - 29° \, 57'$	86°	$x + 5° \, 20'$
80°	$x - 24° \, 23'$	86° 30′	$x + 7° \, 27'$
82° 30′	$x - 11° \, 32'$	88°	$x + 13° \, 31'$

So that if we denote by i_0 that angle of incidence for which θ changes sign, the values of x corresponding to various hypothetical values of i_0 are as follows:—

i_0	75°	78° 54′	80°	82° 30′	85°	86°	86° 30′	88°
x	137° 37′	119° 57′	114° 23′	101° 32′	89° 17′	84° 40′	82° 33′	76° 29′

And if KUNDT's observation be accurate, the value of x is about 103°.

18. From the preceding paragraphs it appears that any value of x lying between 69° 17′ and 82° 22′ will account very well for the four KERR experiments (in the case of iron), except that the agreement with KUNDT's result in the fourth experiment would be imperfect to the extent of four or five degrees.

19. The KERR experiments were also tried on mirrors of nickel and of cobalt, but the observations made were so indefinite that they are of little use as a test of the present theory. In the case of polar reflection from nickel, when the incident light is polarised perpendicularly to the plane of incidence, KUNDT found that the rotation changes sign for an angle of incidence somewhere between 50° and 60°. I have calculated (for yellow light) the values of the vector angle of $\mathrm{B}\cos i/\mathrm{A}$ for these angles of incidence, and find them to be

$$x - 67^\circ\ 12' \quad \text{for} \quad i = 50^\circ,$$
$$x - 57^\circ\ 12' \quad \text{for} \quad i = 60^\circ,$$

so that any value of x lying between 147° 12′ and 157° 12′ will give a satisfactory explanation of this experiment. Also C_0 would be negative.

When the reflection is polar and the incident light is polarised in the plane of incidence, KUNDT finds that θ has the opposite sign to γ_0 for all angles of incidence. The vector angle of $\mathrm{A}/\mathrm{B}\cos i$ is found to be

$$x + 36^\circ\ 42' \quad \text{when} \quad i = 90^\circ,$$

and

$$x + 98^\circ\ 26' \quad \text{when} \quad i = 0^\circ.$$

The cosine of this vector angle will be negative for all angles of incidence, provided the value of x lie between 53° 18′ and 171° 34′.

The two experiments indicate that for nickel x has a value intermediate between 147° and 157°.

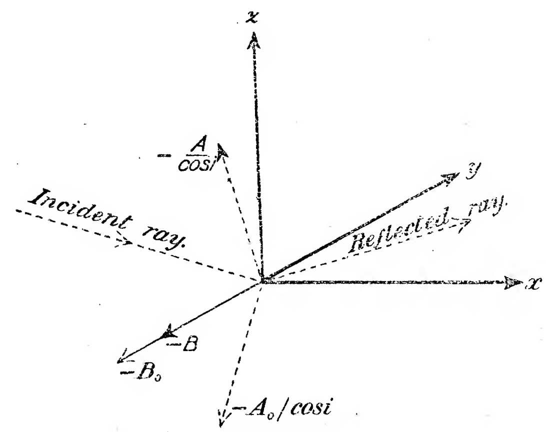
The Experiments of SISSINGH *and* ZEEMAN.

20. I now pass to a much more precise and accurate test of the present theory, the materials for which are to be found in the elaborate series of experiments made during the last few years at Leyden, by SISSINGH and by ZEEMAN.

The first of these series is described by Sissingh in a paper in the 'Archives Néerlandaises,' vol. 27. The experiments were made with an iron mirror, magnetised parallel to the reflecting surface; the amplitude μ and phase m of what Sissingh calls "the magneto-optic component of the reflected light" were measured for various angles of incidence.

One result of the observations is the conclusion that $\mu_i = \mu_p$, and $m_i = m_p$; that is, that for any given angle of incidence the magneto-optic component has the same amplitude and phase, whether the incident light be polarised in or perpendicularly to the plane of incidence. It is to be noticed that the phase of this component is defined as its retardation of phase calculated relatively to that component of ordinary metallic reflection which is polarised in the plane of incidence. The amplitude is reckoned on the supposition that the amplitude of the incident ray is unity.

The accompanying figure shews the relation between the axes of coordinates and the principal directions for the incident and reflected rays, as defined by Sissingh. The standard ray for phase is $-$ B. The standard ray for amplitude is $-\ \mathrm{A}_0/\cos i$, or $-\ \mathrm{B}_0$, according as the incident light is polarised perpendicularly to or in the plane of incidence.



It will be convenient to denote by ϑ the *acceleration* of phase of the magneto-optic component of the reflected ray calculated relatively to that component of ordinary metallic reflection which is polarised in the plane of incidence. If ϑ be calculated from theory, and m from experiment, the theory and experiment will agree if $\vartheta + m = 0°$ or $360°$

21. When the incident light is polarised in the plane of incidence, $\mathrm{A}_0 = 0$, and in formula (36) the incident ray is represented by $-\ \mathrm{B}_0$, the magneto-optic component of the reflected ray by $-\ \mathrm{A}/\cos i$, and the component relatively to which phase is to be measured, by $-$ B. Hence

$$\vartheta_i = \text{vector angle of } \{\mathrm{A}/\mathrm{B}\cos i\}_{\mathrm{A}_0=0}$$

$$= \text{vector angle of } \left.\frac{2.\iota.c\lambda\,(m/\mathrm{M})\,\mathrm{C}_0 e^{\iota x}\,(l\alpha_0 - \mathrm{M}\gamma_0)}{\cos i\,(\mathrm{R}^2 e^{2\iota\alpha} + \mathrm{M}/m)\,(\mathrm{M} - m)}\right\} \quad . \quad . \quad . \quad . \quad . \quad (40).$$

When the incident light is polarised perpendicularly to the plane of incidence, $B_0 = 0$, and the incident ray is represented by $-A_0/\cos i$; the magneto-optic component of the reflected ray by $-B$, that is to say that term in $-B$ which contains the factor A_0. The ray relatively to which phase is measured is represented by that term in $-B$ which contains the (vanishing) factor B_0. If B_0 be supposed to be only just not zero, then, since the incident ray is supposed to be plane polarised, B_0/A_0 is a real quantity. Hence we have

$$\vartheta_p = \text{vector angle of } \left.\frac{2\iota c\lambda\,(\omega^2/\mathrm{M}m)\,\mathrm{C}_0 e^{\iota x}\,(l\alpha_0 + \mathrm{M}\gamma_0)}{(\mathrm{R}^2 e^{2\iota a} + \mathrm{M}/m)\,(\mathrm{M} - m)}\right\} \quad . \quad . \quad . \quad . \quad (41).$$

From (40) and (41) we see at once that when the reflection is equatorial, that is, when $\gamma_0 = 0$,

$$\vartheta_i = \vartheta_p = \vartheta \text{ (say)},$$

and this agrees with SISSINGH's observations.

We also see that when the reflection is polar, that is when $\alpha_0 = 0$,

$$\vartheta_i = \vartheta_p \pm 180^\circ.$$

Now ZEEMAN, as a result of experiments on polar reflection described by him in the 'Archives Néerlandaises,' vol. 27, came to the conclusion that $m_i = m_p$. It is very possible that this discrepancy is due to his using a definition of m_i and m_p slightly different from SISSINGH's.

22. When the reflection is equatorial, we see from (40) that

$$\vartheta = \text{vector angle of } \frac{-2c\lambda \sin i\, \mathrm{C}_0 \alpha_0\, \iota . e^{\iota x} \cos i}{\mathfrak{M}\mathrm{R}^2 e^{2\iota a}\,(\mathfrak{M} - \cos i)\,(\mathfrak{M}\mathrm{R}^{-2} e^{-2\iota a} + \cos i)}.$$

In determining ϑ from this expression there is an ambiguity to the extent of 180°; for in defining m (or $360^\circ - \vartheta$) SISSINGH requires that it shall not be altered when α_0 changes sign. Examining his paper, we see that in equatorial reflection the standard case is when α_0 is negative. Hence, remembering that C_0 is negative, we find that

$$\vartheta = x - 90^\circ - 2a - \text{the sum of the vector angles of}$$
$$\left[\mathfrak{M},\ (\mathfrak{M} - \cos i), \text{ and } (\mathfrak{M}\mathrm{R}^{-2} e^{-2\iota a} + \cos i)\right] . \quad . \quad (42);$$

and to get ϑ accurately for any particular angle of incidence, these three vector angles must be calculated.

The following table shews the results of SISSINGH's observations on the phase for

various angles of incidence, and the theoretical values of the phase for the same angles of incidence, calculated from the present theory.

It is to be observed that the calculation involves only the ordinary optic constants of the metal, and that it is from the comparison with experiment that we derive information as to the value of x in the magneto-optic constant $C_0e^{\iota x}$. There is thus no question of being able to adjust two coefficients, C_0 and x, so as to satisfy the observations, as might be supposed; only one coefficient x is involved, and the test is accordingly a severe one.

EQUATORIAL Reflection from Iron. Yellow Light. $\alpha_0 = -1400$ C.G.S.

Angle of incidence.	Calculated value of $\vartheta - x + 180°$.	SISSINGH's observed value of $m - 180°$.	$\vartheta + m - x$.
86° 0′	267° 25′	29° 26′	296° 51′
82° 30′	274° 41′	24° 22′	299° 3′
76° 30′	283° 29°	14° 49′	298° 18′
71° 25′	288° 47′	10° 3′	298° 50′
61° 30′	295° 42′	1° 49′	297° 31′
51° 22′	300° 12′	− 1° 0′	299° 12′
36° 10′	304° 33′	− 5° 51′	298° 42′
24° 16′	..	doubtful	
12° 0′	..	doubtful	
6° 0′	..	doubtful	

The constancy of the angles in the last column is remarkably good; and the theory accounts for the phenomena with great accuracy if the value assigned to x be the mean of the amounts by which these angles respectively fall short of 360°, namely

$$x = 61° \, 39'.$$

23. When the reflection is polar, we see from (41) that

$$\vartheta_p = \text{vector angle of } \frac{2\mathrm{C}\lambda c_0\gamma_0\iota \, . \, e^{\iota x}}{\mathrm{R}^2e^{2\iota\alpha}\,(\mathfrak{M} - \cos i)\,(\mathfrak{M}\mathrm{R}^{-2}e^{-2\iota\alpha} + \cos i)}\,.$$

Taking γ_0 positive as the standard case, and remembering that C_0 is negative, we find that

$$\vartheta_p = x - 90° - 2\alpha - \text{the sum of the vector angles of } \left[(\mathfrak{M} - \cos i) \text{ and } (\mathfrak{M}\mathrm{R}^{-2}e^{-2\iota\alpha} + \cos i)\right] \quad . \quad . \quad . \quad . \quad (43).$$

Experiments as to the amplitude and phase of the magneto-optic component of light reflected from an iron mirror, magnetised normally to the reflecting surface, have been made by ZEEMAN. He gives an account of these in the 'Archives Néerlandaises,' vol. 27; he confines himself to one angle of incidence, viz., $i = 51° \, 22'$.

His result as regards phase compares with theory as follows :—

POLAR Reflection from Iron. Yellow Light. $\gamma_0 = + 850$ C.G.S.

Angle of incidence.	Calculated value of $\vartheta_p - x + 180°$.	ZEEMAN's observed value of $m - 180°$.	$\vartheta_p + m - x$.
51° 22′	245° 30′	49° 55′	295° 25′

The theory will agree accurately with the experiment if the value of x be

$$x = 64° \ 35'.$$

The values of m given by ZEEMAN for yellow light are

(1.) Derived from "rotations to zero"

$$m_i = 48° \ 58' + 180° \qquad m_p = 50° \ 53' + 180°.$$

(2.) Derived from "minimum rotations"

$$m_i = 45° + 180° \qquad m_p = 44° \ 53' + 180°.$$

In determining the phase the method of "rotations to zero" is preferable to that of "minimum rotations," and so in the above table I have used the mean of the values got by the former method.

24. Another test of the present theory is afforded by observations of the amplitude of the "magneto-optic component." This is denoted by μ_i or μ_p, according as the incident light is polarised in or perpendicularly to the plane of incidence. In the former case the magneto-optic component is represented by $-\mathrm{A}/\cos i$, and the incident ray by $-\mathrm{B}_0$; in the latter case the incident ray is represented by $-\mathrm{A}_0/\cos i$, and the magneto-optic component by $-\mathrm{B}_0$. Hence

$$\mu_i = \mathrm{mod}\left(\frac{\mathrm{A}}{\mathrm{B}_0 \cos i}\right)_{\mathrm{A}_0 = 0} \qquad \mu_p = \mathrm{mod}\left(\frac{\mathrm{B}\cos i}{\mathrm{A}_0}\right)_{\mathrm{B}_0 = 0} \quad . \quad . \quad . \quad . \quad (44).$$

Thus, for *equatorial* reflection, we readily derive from (36)

$$\mu_i = \mathrm{mod}\,\frac{2c\lambda \sin i \cos i \ \mathrm{C}_0 \alpha_0 \iota e^{\iota x}}{\mathfrak{M}\mathrm{R}^2 e^{2\iota a}\,(\mathfrak{M} + \cos i)\,(\mathfrak{M}\mathrm{R}^{-2} e^{-2\iota a} + \cos i)},$$

$$\mu_p = \text{the same},$$

and therefore

$$\mu_i = \mu_p = \mu \ (\text{say}),$$

which agrees with SISSINGH's result.

If for brevity we put $2c\lambda C_0\alpha_0/R^2 \equiv L$, we have

$$\mu = L \,.\, \text{mod}\, \frac{\cos i \sin i}{\mathfrak{M}(\mathfrak{M} + \cos i)(\mathfrak{M}R^{-2}e^{-2\iota\alpha} + \cos i)},$$

and the latter factor may be calculated for any angle of incidence.

In the following table the theoretical values of μ for various angles of incidence are compared with the values observed by SISSINGH. Here again the theoretical value of μ involves the magneto-optic constant $C_0e^{\iota x}$ only by being proportional to C_0, and x is not involved: thus we have not available any adjustment of x to improve the agreement, and the test is very severe.

In fact one set of experiments involves C_0 only, and the other set x only: so that a complex magneto-optic constant really gives no more opportunity for adjustment than would a real one.

EQUATORIAL Reflection from Iron. Yellow Light. $\alpha_0 = -1400$ C.G.S.

Angle of incidence.	Calculated value of $\log_{10}\mu - \log_{10}$ L.	SISSINGH'S observed value of $10^3 \times \mu$.	$\left(\dfrac{\text{Calculated value of } \mu/L}{\text{Observed value of } \mu}\right)$.
86° 0′	$\bar{2}$·1506	·284	49·81
82° 30′	$\bar{2}$·3513	·530	42·37
76° 30′	$\bar{2}$·4916	·715	43·38
71° 25	$\bar{2}$·5397	·815	42·51
61° 30′	$\bar{2}$·5634	·820	44·63
51° 22′	$\bar{2}$·5373	·760	45·34
36° 10′	$\bar{2}$·4305	·630	42·78
24° 16′	$\bar{2}$·3577	·430	52·99
12° 0′	$\bar{3}$·9834	·260	37·02
6° 0′	$\bar{3}$·6849	·125	38·73

Exact agreement of theory with experiment would be indicated by the numbers' in the last column being all equal. Though this is not the case, their approximation to equality is, considering the probability of errors in the observations, remarkably good. The mean of these numbers is about 44; and if we assume C_0 to have such a value that $L = \frac{1}{44}$, the ratios of the calculated to the observed amplitudes for the above angles of incidence taken in order are 1·13, 0·96, 0·99, 0·97, 1·01, 1·03, 0·97, 1·20, 0·84, and 0·88 respectively.

The corresponding value of $-C_0$ is

$$-C_0 = \frac{R^2}{44 \times 2c\lambda \times 1400}$$

where

$$\lambda = 589·6\,.\,10^{-7}, \qquad R^2 = 15·86, \qquad c = 3\,.\,10^{10},$$

the units being electromagnetic and C.G.S. And hence

$$\log_{10}(-\mathrm{C}_0) = \overline{11}\cdot 8623,$$
$$-\mathrm{C}_0 = 7\cdot 283 \times 10^{-11}.$$

25. For *polar* reflection, we derive from (44) and (36)

$$\mu_p = \text{mod.}\, \frac{2c\lambda \cos i\,(-\mathrm{C}_0)\,\gamma_0 \iota\, .\, e^{\iota\varkappa}}{\mathrm{R}^2 e^{2\iota\alpha}\,(\mathfrak{M} + \cos i)\,(\mathfrak{M}\mathrm{R}^{-2}e^{-2\iota\alpha} + \cos i)},$$
$$= -\mu_i, \qquad = \mu \text{ (say)}.$$

Comparing this with the amplitude in equatorial reflection, we find

$$\frac{\mu\,(\text{equatorial})}{\mu\,(\text{polar})} = \text{mod.}\, \frac{(-\alpha_0)\sin i}{\mathfrak{M}\gamma_0}.$$

If $-\alpha_0 = 1400$, $\gamma_0 = 850$, $i = 51^\circ\ 22'$, the value of this ratio, as calculated from theory, is ·321.

But the values ascribed to α_0, γ_0, and i correspond to the experiments of SISSINGH and ZEEMAN; and the latter found experimentally

$$\frac{\mu\,(\text{Sissingh})}{\mu\,(\text{Zeeman})} = \cdot 294.$$

So that here again we have a very fair agreement of the theory with experiment.

Nickel

26. In the paper already quoted ZEEMAN gives a few measurements made by himself on polar reflection from nickel. He also quotes experimental results of KUNDT ('Wied. Ann.,' vol. 23), and DRUDE ('Wied. Ann.,' vol. 46), which he expresses in a form similar to his own. These I have used to form the following tables, wherein the theoretical values of the phase and amplitude are in all cases calculated for yellow light.

EQUATORIAL Reflection from Nickel. (Probably) White Light. $\alpha_0 =$

Angle of incidence.	Calculated value of $\vartheta - x + 180°$.	KUNDT's observed value of $m_i - 180°$.	$\vartheta + m_i - x$.
30° 6′	337° 17′	− 3° 50′	333° 27′
40°	334° 48′	−64° 18′	270° 30′
50°	331° 15′	−64° 46′	266° 29′
61° 30′	325° 11′	−52° 21′	272° 50′
65° 18′	322° 27′	−53° 18′	269° 9′
75°	312° 43′	−49° 54′	262° 49′

Fairly good agreement is here indicated (except in case of first angle of incidence) if the value of x be about

$$x = 91° 30'.$$

EQUATORIAL Reflection from Nickel. White Light. $\alpha_0 =$

Angle of incidence.	Calculated value of $\vartheta - x + 180°$.	DRUDE's observed value of $m - 180°$.	$\vartheta + m - x$.
60°	326° 9′	−48° 22′	277° 47′
65°	322° 40′	−46° 3′	276° 37′
75°	312° 43′	+11° 41′	324° 24′
80°	305° 11′	− 8° 42′	296° 29′

If $x = 76° 30'$ or thereabouts a fairly good agreement is indicated, except in the case of $i = 75°$. For this case DRUDE's observation differs widely from KUNDT's, and is perhaps wrong.

EQUATORIAL Reflection from Nickel.

Angle of incidence.	Calculated value of $\log_{10}\mu - \log_{10}L$.	KUNDT's observed value of $10^3 \times \mu$.	$\left(\frac{\text{Calculated value of } \mu/L}{\text{Observed value of } \mu}\right)$.
30° 6′	$\bar{2}$·4193	·21	125·1
40°	$\bar{2}$·5205	·77	43·05
50°	$\bar{2}$·5842	1·39	27·62
61° 30′	$\bar{2}$·6139	·90	45·68
65° 18′	$\bar{2}$·6111	·84	48·62
75°	$\bar{2}$·5635	·23	159·1

The agreement of theory with experiment is here more defective. As the intensity

of magnetisation is not stated, this series of experiments gives no information as to the value of C_0.

POLAR Reflection from Nickel. Yellow Light. $\gamma_0 =$

Angle of incidence.	Calculated value of $\vartheta_p + 180° - x$.	ZEEMAN's observed value of $m - 180°$.	$\vartheta_p + m - x$.
50°	268° 33′	11° 40′	280° 13′

shewing agreement if $x = 79° \ 47'$.

The experiments quoted in the two following tables are from a paper of ZEEMAN's ('Communications from the Leiden Laboratory of Physics,' No. 10) :—

POLAR Reflection from Nickel. White Light. $\gamma_0 = 2190$ C.G.S.

Angle of incidence.	Calculated value of $\vartheta_i - x$.	Observed value of m.	$\vartheta_i + m - x$.
25° 39° 4′	276° 13′ 272° 43′	5° 9′ 9° 17′	281° 22′ 282°

shewing agreement if $x = 78° \ 19'$.

POLAR Reflection from Nickel. White Light. $\gamma_0 = 2190$ C.G.S.

Angle of incidence.	Calculated value of $\log_{10}(-\mu_i) - \log_{10} L'$.	ZEEMAN's observed value of $10^3 \times \mu$.	$\left(\dfrac{\text{Calculated value of } \mu/L'}{\text{Observed value of } \mu}\right)$.
39° 4′ 25°	$\bar{1}\cdot2940$ $\bar{1}\cdot3000$	− ·975 − 1·00	201·8 199·5

wherein $L' \equiv 2c\lambda(-C_0)\gamma_0/R^2$.

The agreement indicated is excellent, provided C_0 have such a value that

$$L' = \tfrac{1}{200},$$

namely,

$$\log(-C_0) = \overline{12}\cdot9649$$
$$-C_0 = 9\cdot225 \times 10^{-12}$$

The experimental results used in the following table are taken from a paper by Dr. C. H. WIND ('Communications from the Leiden Laboratory of Physics,' No. 9) :—

POLAR Reflection from Nickel. Yellow Light.

Angle of incidence.	Strength of magnetic field in C.G.S. units.	Calculated value of $\vartheta_i - x$.	Observed value of m_i.	$\vartheta_i + m_i - x$.
39° 4′	2190	272° 43′	14° 32′	287° 15′
55°	9560	266° 7′	17° 47′	283° 54′
75°	12470	249° 28′	32° 25′	281° 53′

For incidence of 39° 4′ ZEEMAN'S result is to be preferred to that of WIND, as he took more precautions to eliminate causes of error. For the other two angles of incidence agreement is indicated if x is about 77°.

In estimating the consistency of the above results it is to be remembered that the optic constants (R and α) for different specimens of nickel are often sensibly different. (I have used the same set of values all through.) Indeed in the case of iron ZEEMAN found that his observations of the optic constants of a particular mirror, made respectively before and after an observation of the KERR phenomena, differed considerably.

Cobalt.

27. In his paper in the 'Archives Néerlandaises,' vol. 27, ZEEMAN describes experiments made by himself on mirrors of cobalt, and also quotes the results of experiments made by DRUDE. The comparison of these with the present theory is shewn in the following tables :—

POLAR Reflection from Cobalt. White Light.

Angle of incidence.	Calculated value of $\vartheta_p - x + 180°$.	ZEEMAN'S observed value of $m - 180°$.	$\vartheta_p + m - x - 360°$.
45°	272° 11′	20° 34′	−67° 15′
60°	265° 34′	27° 40′	−66° 46′
73°	255° 6′	37° 55′	−66° 59′

Good agreement is indicated if $x = 67°$.

POLAR Reflection from Cobalt. Green Light.

Angle of incidence.	Calculated value of $\vartheta_p - x + 180°$.	ZEEMAN'S observed value of $m - 180°$.	$\vartheta_p + m - x - 360°$.
50°	270° 23′	25° 9′	−64° 28′
60°	265° 34′	32° 30′	−61° 56′
72°	256° 13′	45° 51′	−57° 56′

This shews fairly good agreement if x is about 61° 30′. In the above ϑ is calculated from the constants for yellow light.

EQUATORIAL Reflection from Cobalt. White Light.

Angle of incidence.	Calculated value of $\vartheta - x + 180°$.	DRUDE'S observed value of $m - 180°$.	$\vartheta + m - x - 360°$.
35°	337° 39′	−77° 24′	−99° 45′
60°	328° 40′	−25° 27′	−56° 47′
75°	315° 59′	−12° 56′	−56° 57′
83°	302° 9′	−12° 57′	−70° 48′

Here the agreement is not so good; the last three angles of incidence would indicate that x is about 61° 30′. DRUDE'S method is that of minimum rotations, wherein errors of observation influence the phase to a much greater extent than in the method of null-rotations.

The following experiments are from a paper of ZEEMAN'S ('Communications from the Leiden Laboratory of Physics,' No. 5):—

POLAR Reflection from Cobalt. White Light. $\gamma_0 = 430$ C.G.S.

Angle of incidence.	Calculated value of $\log_{10}(\mu_p) - \log_{10} L'$.	ZEEMAN'S observed value of $10^3 \times \mu$.	$\left(\frac{\text{Calculated value of } \mu/L'}{\text{Observed value of } \mu}\right)$.
45°	$\bar{1}$·2333	1·58	108·3
60°	$\bar{1}$·2092	1·50	107·9
73°	$\bar{1}$·1413	1·17	118·3

wherein $L' = 2c\lambda(-C_0)\gamma_0/R^2$.

Here the agreement is very good, the indicated value of L′ being about $\frac{1}{111}$; if this be so, we have for cobalt

$$\log(-C_0) = \overline{10}\cdot0889,$$

i.e.,

$$-C_0 = 1\cdot227 \times 10^{-10}.$$

The Hall Effect.

28. Before leaving this part of the subject it is worth while to investigate whether the ordinary HALL effect is large enough to contribute, to any appreciable extent, to the phenomena we have been considering.

If $\mathfrak{E}$ be HALL'S constant, as usually defined, the equations into which it enters are of the type

$$P = P' + \mathfrak{E}(\beta_0'w - \gamma_0'v),$$

where $(\alpha_0', \beta_0', \gamma_0')$ is the magnetic force. Comparing this with the form that equations (11) would assume if (b_1, b_2, b_3) were zero, it appears that

$$\mathfrak{E}\beta_0' = H^2 g_2 = -\frac{4c^2\lambda^2}{R^4e^{4\iota\alpha}}g^2;$$

so that

$$g_2 = -\mathfrak{E}\frac{R^4e^{4\iota\alpha}}{4c^2\lambda^2}\beta_0'.$$

Now for iron,

$$\mathfrak{E} = 7850 \times 10^{-15},$$

and if we substitute the values of R, α, λ, corresponding to yellow light, we find

$$g_2 = -\beta_0' \,.\, Q \,.\, e^{\iota(145^\circ\,20')}$$

where $\log_{10} Q = \overline{16}\cdot4466$.

But we see from § 24 that $\eta_2 = \beta_0 \,.\, C_0e^{\iota x}$, where $\log_{10}(-C_0) = \overline{11}\cdot8623$. Also $\beta_0 = \mu\beta_0'$, where μ is the magnetic permeability of iron and is greater than unity.

Hence the modulus of the fraction g_2/η_2 has a logarithm less than $\overline{6}\cdot5843$, so that the modulus itself is less than $\frac{1}{260000}$. Thus it appears that the ordinary HALL effect is more than two hundred thousand times too small to account for the KERR phenomena.

But, in order that the coefficients (b_1, b_2, b_3) should be real, it is necessary that the imaginary parts of the complexes (η_1, η_2, η_3) should be supplied by (g_1, g_2, g_3), that is to say by the HALL effect. Hence it must be concluded that the coefficient of the HALL effect is very much greater for excessively rapidly alternating currents than

for steady ones. There is nothing unnatural in this, for the incipient conductions which make optical opacity have no relation of continuity whatever with the steady conduction in an ordinary current; thus MAXWELL found that the ordinary coefficients of "conductivity" are very much smaller in the optical circumstances. And it may be noticed that, as $\mathfrak{C}$ is proportional to electromotive force divided by current, a greatly diminished conductivity will correspond to a greatly increased value of $\mathfrak{C}$.

The value which HALL's constant would, on this supposition, have for yellow light, is obtainable from the equation

$$\text{Imaginary part of } \eta_2 = \text{imaginary part of } -\mathfrak{C}\frac{R^4 e^4_{\iota\alpha}}{4c^2\lambda^2}\beta_0$$

wherein I now take the HALL effect to be proportional, not to the magnetic force, but to the intensity of magnetisation.

This gives

$$\mathfrak{C} = -\frac{4c^2\lambda^2}{R^4 \sin 4\alpha} C_0 \sin x$$

$$= +(5\cdot670) \text{ for iron, and yellow light.}$$

The real part of g_2 is then $C_0 \sin x \cot 4\alpha . \beta_0$, so that, if $(b_1, b_2, b_3) = E_0(\alpha_0, \beta_0, \gamma_0)$,

$$-p^2 E_0 = C_0 \cos x - C_0 \sin x \cot 4\alpha$$

$$= -C_0 \frac{\sin(x - 4\alpha)}{\sin 4\alpha}$$

and hence we find

$$\log_{10} E_0 = \overline{41}\cdot 0939, \qquad E_0 = (1\cdot 242) . 10^{-41}.$$

Effect of Magnetisation Perpendicular to the Plane of Incidence.

29. A very interesting inference from the presence of η_2 in the equations (36) is that, if the present theory be true, the component of magnetisation perpendicular to the plane of incidence will produce an effect not quite the same as the KERR phenomenon, but of the same order of magnitude.

On enquiring whether such an effect had ever been observed or measured, I found that a few months ago it was predicted from theoretical considerations by Dr. C. H. WIND, in a paper which has as yet appeared only in Dutch. Acting on this prediction ZEEMAN sought the phenomenon experimentally, found it, and succeeded in measuring it. His results are published in the 'Communications from the Leiden Laboratory of Physics,' No. 29.

Let us suppose that the magnetisation is entirely perpendicular to the plane of incidence; then $\eta_1 = 0$ and $\eta_3 = 0$, and the reflected light is specified by A and B, where, from equations (36),

$$A = \frac{-\left\{R^2e^{2\iota\alpha} - \frac{M}{m}\left(1 + \iota\frac{2c\lambda}{R^2e^{2\iota\alpha}}\frac{l}{M}\eta_2\right)\right\}}{\left\{R^2e^{2\iota\alpha} + \frac{M}{m}\left(1 + \iota\frac{2c\lambda}{R^2e^{2\iota\alpha}}\frac{l}{M}\eta_2\right)\right\}} A_0,$$

$$B = \frac{m - M}{m + M} B_0.$$

From these expressions we see that, if the incident ray is polarised in the plane of incidence, so that $A_0 = 0$, the expression for the reflected ray does not involve η_2; and so the magnetisation β_0 produces no effect. This is in agreement with the prediction of WIND.

But if the incident ray be polarised perpendicularly to the plane of incidence, so that $B_0 = 0$, the reflected ray is given by A, which does contain η_2. Instead of having

$$A = \frac{-(R^2e^{2\iota\alpha} - M/m)}{(R^2e^{2\iota\alpha} + M/m)}$$

as would be the case if there were no magnetisation, we have A equal to this value multiplied by the factor

$$\frac{1 - \{\iota.2c\lambda R^{-2}e^{-2\iota\alpha}(l/m)\,C_0e^{\iota x}\beta_0\}/\{R^2e^{2\iota\alpha} - M/m\}}{1 + \{\iota.2c\lambda R^{-2}e^{-2\iota\alpha}(l/m)\,C_0e^{\iota x}\beta_0\}/\{R^2e^{2\iota\alpha} + M/m\}},$$

which is the same as

$$1 + C_0\beta_0.2c\lambda.\sin 2i.\frac{\iota.e^{\iota.x}}{R^4e^{4\iota\alpha}(\cos i - \mathfrak{M}R^{-2}e^{-2\iota\alpha})(\cos i + \mathfrak{M}R^{-2}e^{-2\iota\alpha})},$$

and the effect of this factor, which of course differs from unity by only a very small quantity, is to slightly alter both the amplitude and the phase of the still plane-polarised reflected ray.

Now the change of phase produced by a factor of the form $1 + Qe^{\iota q}$, where Q is very small, is $\tan^{-1}\{Q \sin q/(1 + Q \cos q)\}$, and therefore, in circular measure, is approximately $Q \sin q$.

Hence, if for brevity we put

$$\cos i - \mathfrak{M}R^{-2}e^{-2\iota\alpha} \equiv Ye^{\iota y}, \qquad \cos i + \mathfrak{M}R^{-2}e^{-2\iota\alpha} \equiv Y'e^{\iota y'},$$

the acceleration of phase produced in the reflected ray by the component β_0 of magnetisation is, in circular measure,

$$C_0.\beta_0.2c\lambda.\frac{\sin 2i}{R^4.Y.Y'}\sin(x + 90^\circ - y - y' - 4\alpha).$$

In ZEEMAN'S experiment the angle of incidence was 75° and the intensity of

magnetisation was a little over 1100 C.G.S., the mirror being of iron. Under these circumstances he found the acceleration of phase to be $\cdot 003 \times 90^\circ$ with a mean error of $\cdot 001 \times 90^\circ$.

Calculating the theoretical value, we notice that

$$\begin{aligned} Y &= \cdot 2249, & y &= -(62^\circ\ 43'), \\ Y' &= \cdot 4601, & y' &= \quad 25^\circ\ 44', \end{aligned}$$

and if we assume $x = 63^\circ$ (which is about the mean of the values indicated by the experiments of SISSINGH and ZEEMAN), then

$$x + 90^\circ - y - y' - 4\alpha = 360^\circ + 44^\circ\ 39';$$

we may also assume

$$\log_{10}(-C_0) = \overline{11}\cdot 8623.$$

With these values we find that the change of phase indicated by the theory is, in circular measure,

$$\cdot 003818,$$

or, in degrees,

$$\cdot 00243 \times 90^\circ.$$

This agrees very well with ZEEMAN's observations.

Transmission through Metal Films.

30. Another effect of the action of magnetism on light is the rotation of the plane of polarisation of normally incident light, on passing through very thin films of magnetised metal. The principal experiments in this subject have been made by KUNDT, DU BOIS, LOBACH, and DRUDE; they found that the rotation is always in the direction of the magnetising current, and measured it in special cases. It is desirable to compare these measurements with the mathematical solution of the problem worked out on the basis of the present theory.

Let the film be bounded by the planes

$$z = 0 \qquad \text{and} \qquad z = -h,$$

and let the incident light fall normally on the surface $z = 0$. The external magnetic field is supposed to be parallel to the axis of z, so that $\alpha_0 = 0$ and $\beta_0 = 0$. The plane of polarisation of the incident light is taken as the plane of yz.

Thus we may assume the following expressions to represent the light in the air on the two sides of the film, and in the film itself.

In the air $(z > 0)$

$$u = A_0 e^{\iota(mz+pt)} + Ae^{\iota(-mz+pt)},$$
$$v = \qquad\qquad Be^{\iota(-mz+pt)},$$
$$w = 0.$$

In the metal $(0 > z > -h)$

$$u = A_1 e^{\iota(m_1z+pt)} + A'_1 e^{\iota(-m_1z+pt)} + A_2 e^{\iota(m_2z+pt)} + A'_2 e^{\iota(-m_2z+pt)},$$
$$v = \iota A_1 e^{\iota(m_1z+pt)} + \iota A'_1 e^{\iota(-m_1z+pt)} - \iota A_2 e^{\iota(m_2z+pt)} - \iota A'_2 e^{\iota(-m_2z+pt)},$$
$$w = 0.$$

In the air $(z + h < 0)$

$$u = Ee^{\iota(mz+pt)}, \qquad v = Fe^{\iota(mz+pt)}, \qquad w = 0,$$

wherein the incident ray is represented by A_0, the reflected ray by (A, B), and the transmitted ray by (E, F).

It should be noticed that in these assumptions multiple reflections are not neglected; all waves in the film are included in the complex constants A_1, A'_1, A_2, and A'_2.

For surface conditions we may establish the continuity of

$$H(u - H\eta_3 v), \quad Hd/dz(u - H\eta_3 v), \quad H(v + H\eta_3 u), \quad \text{and} \quad Hd/dz(v + H\eta_3 u)$$

respectively; the second of these is the expression of the continuity of the magnetic force b, and is used instead of the continuity of w to which it is equivalent, as it leads to more symmetrical analysis.

At the surface $z = 0$ these boundary conditions lead to the equations

$$H(A_0 + A) = H'(A_1 + A'_1 + A_2 + A'_2) - H'^2\iota\eta_3(A_1 + A'_1 - A_2 - A'_2),$$

$$Hm(A_0 - A) = H'(m_1A_1 - m_1A'_1 + m_2A_2 - m_2A'_2)$$
$$- H'^2\iota\eta_3(m_1A_1 - m_1A'_1 - m_2A_2 + m_2A'_2),$$

$$-H\iota B = H'(A_1 + A'_1 - A_2 - A'_2) - H'^2\iota\eta_3(A_1 + A'_1 + A_2 + A'_2),$$

$$-Hm\iota B = H'(-m_1A_1 + m_1A'_1 + m_2A_2 - m_2A'_2)$$
$$+ H'^2\iota\eta_3(m_1A_1 - m_1A'_1 + m_2A_2 - m_2A'_2).$$

At the surface $z = -h$, (if for brevity we write $-\iota m_1 h = \theta$, and $-\iota m_2 h = \phi$), the boundary conditions lead to the equations

$$\mathrm{HE}e^{-\iota mh} = \mathrm{H}'\left(\mathrm{A}_1e^{\theta} + \mathrm{A}_1'e^{-\theta} + \mathrm{A}_2e^{\phi} + \mathrm{A}_2'e^{-\phi}\right)$$
$$- \mathrm{H}'^2\iota\eta_3\left(\mathrm{A}_1e^{\theta} + \mathrm{A}_1'e^{-\theta} - \mathrm{A}_2e^{\phi} - \mathrm{A}_2'e^{-\phi}\right),$$

$$\mathrm{H}m\mathrm{E}e^{-\iota mh} = \mathrm{H}'\left(m_1\mathrm{A}_1e^{\theta} - m_1\mathrm{A}'_1e^{-\theta} + m_2\mathrm{A}_2e^{\phi} - m_2\mathrm{A}_2'e^{-\phi}\right)$$
$$- \mathrm{H}'^2\iota\eta_3\left(m_1\mathrm{A}_1e^{\theta} - m_1\mathrm{A}_1'e^{-\theta} - m_2\mathrm{A}_2e^{\phi} + m_2\mathrm{A}_2'e^{-\phi}\right),$$

$$- \mathrm{H}\iota\mathrm{F}e^{-\iota mh} = \mathrm{H}'\left(\mathrm{A}_1e^{\theta} + \mathrm{A}_1'e^{-\theta} - \mathrm{A}_2e^{\phi} - \mathrm{A}'_2e^{-\phi}\right)$$
$$- \mathrm{H}'^2\iota\eta_3\left(\mathrm{A}_1e^{\theta} + \mathrm{A}_1'e^{-\theta} + \mathrm{A}_2e^{\phi} + \mathrm{A}_2'e^{-\phi}\right),$$

$$\mathrm{H}m\iota\mathrm{F}e^{-\iota mh} = \mathrm{H}'\left(- m_1\mathrm{A}_1e^{\theta} + m_1\mathrm{A}_1'e^{-\theta} + m_2\mathrm{A}_2e^{\phi} - m_2\mathrm{A}_2'e^{-\phi}\right)$$
$$+ \mathrm{H}'^2\iota\eta_3\left(m_1\mathrm{A}_1e^{\theta} - m_1\mathrm{A}_1'e^{-\theta} + m_2\mathrm{A}_2e^{\phi} - m_2\mathrm{A}_2'e^{-\phi}\right).$$

If, for brevity, we put $\mathrm{H}'\iota\eta_3 \equiv t$, and notice from equations (25) that in the present problem

$$m_1/\mathrm{M} = 1 + \tfrac{1}{2}t, \qquad m_2/\mathrm{M} = 1 - \tfrac{1}{2}t,$$

we readily reduce the boundary conditions to the following:—

$$\frac{\mathrm{H}}{\mathrm{H}'}(\mathrm{A}_0 + \mathrm{A}) = \mathrm{A}_1 + \mathrm{A}_1' + \mathrm{A}_2 + \mathrm{A}_2' - t\left(\mathrm{A}_1 + \mathrm{A}_1' - \mathrm{A}_2 - \mathrm{A}_2'\right)$$

$$\frac{\mathrm{H}}{\mathrm{H}'}\frac{m}{\mathrm{M}}(\mathrm{A}_0 - \mathrm{A}) = \mathrm{A}_1 - \mathrm{A}_1' + \mathrm{A}_2 - \mathrm{A}_2' - \tfrac{1}{2}t\left(\mathrm{A}_1 - \mathrm{A}_1' - \mathrm{A}_2 + \mathrm{A}_2'\right)$$

$$-\frac{\mathrm{H}}{\mathrm{H}'}\iota\mathrm{B} = \mathrm{A}_1 + \mathrm{A}_1' - \mathrm{A}_2 - \mathrm{A}_2' - t\left(\mathrm{A}_1 + \mathrm{A}_1' + \mathrm{A}_2 + \mathrm{A}_2'\right)$$

$$\frac{\mathrm{H}}{\mathrm{H}'}\frac{m}{\mathrm{M}}\iota\mathrm{B} = \mathrm{A}_1 - \mathrm{A}_1' - \mathrm{A}_2 + \mathrm{A}_2' - \tfrac{1}{2}t\left(\mathrm{A}_1 - \mathrm{A}_1' + \mathrm{A}_2 - \mathrm{A}_2'\right),$$

$$\frac{\mathrm{H}}{\mathrm{H}'}\mathrm{E}e^{-\iota mh} = \mathrm{A}_1e^{\theta} + \mathrm{A}_1'e^{-\theta} + \mathrm{A}_2e^{\phi} + \mathrm{A}_2'e^{-\phi} - t\left(\mathrm{A}_1e^{\theta} + \mathrm{A}_1'e^{-\theta} - \mathrm{A}_2e^{\phi} - \mathrm{A}_2'e^{-\phi}\right)$$

$$\frac{\mathrm{H}}{\mathrm{H}'}\frac{m}{\mathrm{M}}\mathrm{E}e^{-\iota mh} = \mathrm{A}_1e^{\theta} - \mathrm{A}_1'e^{-\theta} + \mathrm{A}_2e^{\phi} - \mathrm{A}_2'e^{-\phi} - \tfrac{1}{2}t\left(\mathrm{A}_1e^{\theta} - \mathrm{A}_1'e^{-\theta} - \mathrm{A}_2e^{\phi} + \mathrm{A}_2'e^{-\phi}\right)$$

$$-\frac{\mathrm{H}}{\mathrm{H}'}\iota\mathrm{F}e^{-\iota mh} = \mathrm{A}_1e^{\theta} + \mathrm{A}_1'e^{-\theta} - \mathrm{A}_2e^{\phi} - \mathrm{A}_2'e^{-\phi} - t\left(\mathrm{A}_1e^{\theta} + \mathrm{A}_1'e^{-\theta} + \mathrm{A}_2e^{\phi} + \mathrm{A}_2'e^{-\phi}\right)$$

$$-\frac{\mathrm{H}}{\mathrm{H}'}\frac{m}{\mathrm{M}}\iota\mathrm{F}e^{-\iota mh} = \mathrm{A}_1e^{\theta} - \mathrm{A}_1'e^{-\theta} - \mathrm{A}_2e^{\phi} + \mathrm{A}_2'e^{-\phi} - \tfrac{1}{2}t\left(\mathrm{A}_1e^{\theta} - \mathrm{A}_1'e^{-\theta} + \mathrm{A}_2e^{\phi} - \mathrm{A}_2'e^{-\phi}\right).$$

Eliminating B, E, and F from the last six, and representing m/M by x, we get

$$\left\{1+x-t\left(\tfrac{1}{2}+x\right)\right\} A_1 - \left\{1-x-t\left(\tfrac{1}{2}-x\right)\right\} A_1'$$
$$- \left\{1+x+t\left(\tfrac{1}{2}+x\right)\right\} A_2 + \left\{1-x+t\left(\tfrac{1}{2}-x\right)\right\} A_2' = 0,$$

$$\left\{1-x-t\left(\tfrac{1}{2}-x\right)\right\} A_1 e^{\theta} - \left\{1+x-t\left(\tfrac{1}{2}+x\right)\right\} A_1' e^{-\theta}$$
$$+ \left\{1-x+t\left(\tfrac{1}{2}-x\right)\right\} A_2 e^{\phi} - \left\{1+x+t\left(\tfrac{1}{2}+x\right)\right\} A_2' e^{-\phi} = 0,$$

$$\left\{1-x-t\left(\tfrac{1}{2}-x\right)\right\} A_1 e^{\theta} - \left\{1+x-t\left(\tfrac{1}{2}+x\right)\right\} A_1' e^{-\theta}$$
$$- \left\{1-x+t\left(\tfrac{1}{2}-x\right)\right\} A_2 e^{\phi} + \left\{1+x+t\left(\tfrac{1}{2}+x\right)\right\} A_2' e^{-\phi} = 0.$$

The last two equations give

$$\frac{A_1 e^{\theta}}{(1+x)-t\left(\frac{1}{2}+x\right)} = \frac{A_1' e^{-\theta}}{(1-x)-t\left(\frac{1}{2}-x\right)} = \xi \text{ (say)}$$
$$\frac{A_2 e^{\phi}}{(1+x)+t\left(\frac{1}{2}+x\right)} = \frac{A_2' e^{-\phi}}{(1-x)+t\left(\frac{1}{2}-x\right)} = \zeta \text{ (say)}$$

and substitution from these in the first leads to

$$\xi\left[e^{-\theta}\left\{(1+x)-t\left(\tfrac{1}{2}+x\right)\right\}^2 - e^{\theta}\left\{(1-x)-t\left(\tfrac{1}{2}-x\right)\right\}^2\right]$$
$$= \zeta\left[e^{-\phi}\left\{(1+x)+t\left(\tfrac{1}{2}+x\right)\right\}^2 - e^{\phi}\left\{(1-x)+t\left(\tfrac{1}{2}-x\right)\right\}^2\right],$$

whence,

$$\frac{A_1 e^{\theta}}{\left\{1+x-t\left(\frac{1}{2}+x\right)\right\}\left[e^{-\phi}\left\{1+x+t\left(\frac{1}{2}+x\right)\right\}^2 - e^{\phi}\left\{1-x+t\left(\frac{1}{2}-x\right)\right\}^2\right]}$$
$$= \frac{A_1' e^{-\theta}}{\left\{1-x-t\left(\frac{1}{2}-x\right)\right\}\left[e^{-\phi}\left\{1+x+t\left(\frac{1}{2}+x\right)\right\}^2 - e^{\phi}\left\{1-x+t\left(\frac{1}{2}-x\right)\right\}^2\right]}$$
$$= \frac{A_2 e^{\phi}}{\left\{1+x+t\left(\frac{1}{2}+x\right)\right\}\left[e^{-\theta}\left\{1+x-t\left(\frac{1}{2}+x\right)\right\}^2 - e^{\theta}\left\{1-x-t\left(\frac{1}{2}-x\right)\right\}^2\right]}$$
$$= \frac{A_2' e^{-\phi}}{\left\{1-x+t\left(\frac{1}{2}-x\right)\right\}\left[e^{-\theta}\left\{1+x-t\left(\frac{1}{2}+x\right)\right\}^2 - e^{\theta}\left\{1-x-t\left(\frac{1}{2}-x\right)\right\}^2\right]}.$$

Substituting from these in the sixth and eighth of the surface conditions, we get

$$-\iota\frac{F}{E} = \frac{X-Z}{X+Z}$$

where

$$X \equiv (1-t)(2-t)\left[e^{-\phi}\left\{1+x+t\left(\tfrac{1}{2}+x\right)\right\}^2 - e^{\phi}\left\{1-x+t\left(\tfrac{1}{2}-x\right)\right\}^2\right]$$

and

$$Z = (1+t)(2+t)\left[e^{-\theta}\left\{1+x-t\left(\tfrac{1}{2}+x\right)\right\}^2 - e^{\theta}\left\{1-x-t\left(\tfrac{1}{2}-x\right)\right\}^2\right].$$

Now

$$\theta = -\iota m_1 h = -\iota \mathrm{M}h(1 + \tfrac{1}{2}t)$$

$$\phi = -\iota m_2 h = -\iota \mathrm{M}h(1 - \tfrac{1}{2}t)$$

and therefore, if the modulus of $\iota\mathrm{M}h$ be not exceedingly great,

$$e^{\theta} = e^{-\iota\mathrm{M}h}(1 - \tfrac{1}{2}t\,.\,\iota\mathrm{M}h) \qquad e^{-\theta} = e^{\iota\mathrm{M}h}(1 + \tfrac{1}{2}t\,.\,\iota\mathrm{M}h)$$

$$e^{\phi} = e^{-\iota\mathrm{M}h}(1 + \tfrac{1}{2}t\,.\,\iota\mathrm{M}h) \qquad e^{-\phi} = e^{\iota\mathrm{M}h}(1 - \tfrac{1}{2}t\,.\,\iota\mathrm{M}h).$$

Substituting these expressions, we readily find that, to the first order in t,

$$\mathrm{X} = e^{\iota\mathrm{M}h}(1 + x)\left\{2(1 + x) - t(1 - x) - t\,.\,\iota\mathrm{M}h(1 + x)\right\}$$
$$- e^{-\iota\mathrm{M}h}(1 - x)\left\{2(1 - x) - t(1 + x) + t\,.\,\iota\mathrm{M}h(1 - x)\right\},$$

$$\mathrm{Z} = e^{\iota\mathrm{M}h}(1 + x)\left\{2(1 + x) + t(1 - x) + t\,.\,\iota\mathrm{M}h(1 + x)\right\}$$
$$- e^{-\iota\mathrm{M}h}(1 - x)\left\{2(1 - x) + t(1 + x) - t\,.\,\iota\mathrm{M}h(1 - x)\right\},$$

and so

$$\iota\frac{\mathrm{F}}{\mathrm{E}} = \frac{t\left[e^{\iota\mathrm{M}h}(1 + x)\left\{1 - x + \iota\mathrm{M}h(1 + x)\right\} - e^{-\iota\mathrm{M}h}(1 - x)\left\{1 + x - \iota\mathrm{M}h(1 - x)\right\}\right]}{2\left\{e^{\iota\mathrm{M}h}(1 + x)^2 - e^{-\iota\mathrm{M}h}(1 - x)^2\right\}}$$

whence

$$\frac{\mathrm{F}}{\mathrm{E}} = \tfrac{1}{2}\mathrm{H}'\eta_3\frac{\left[\left\{\frac{1 - x}{1 + x} + \iota\mathrm{M}h\right\} - \left\{\frac{1 - x}{1 + x} - \left(\frac{1 - x}{1 + x}\right)^2\iota\mathrm{M}h\right\}e^{-2\iota\mathrm{M}h}\right]}{\left[1 - \left(\frac{1 - x}{1 + x}\right)^2 e^{-2\iota\mathrm{M}h}\right]},$$

and if θ be the angle through which the major axis of the ellipse of polarisation of the transmitted light is rotated from the axis of x towards the axis of y, θ is the real part of F/E. For a given metal, and given values of h and λ, this angle can be calculated exactly from the above formula, but such calculation would be very tedious. It is well therefore to examine the relative magnitudes of the different terms in cases corresponding to the known experiments on this subject, in order to see whether there is any approximate formula of a simpler character. It will be sufficient to consider some of the experiments on transmission through films of *iron* described by LOBACH ('Wied. Ann.' vol. 39, p. 356) and by DRUDE ('Wied. Ann.' vol. 46, p. 416).

Now

$$\frac{1 - x}{1 + x} = \frac{\mathrm{R} - \cos\alpha + \iota\sin\alpha}{\mathrm{R} + \cos\alpha - \iota\sin\alpha}$$

$$= \left(\frac{\mathrm{R}^2 + 1 - 2\cos\alpha}{\mathrm{R}^2 + 1 + 2\cos\alpha}\right)^{\frac{1}{2}} e^{\iota\tan^{-1}\left\{2\mathrm{R}\sin\alpha/(\mathrm{R}^2 - 1)\right\}}$$

so that for iron

$$(1-x)/(1+x) = (\cdot 7500)\, e^{-\iota (23^\circ 21')}$$
$$= (\cdot 6887) - \iota (\cdot 2973).$$

Also

$$-2\iota \mathrm{M}h = -2\iota (\cos\alpha + \iota \sin\alpha)\, \mathrm{R}mh$$
$$= (\sin\alpha - \iota\cos\alpha)\, \mathrm{R}h\, (4\pi/\lambda)$$
$$= -4\pi\, \{(3\cdot 21) + \iota (2\cdot 36)\}\, h/\lambda.$$

In DRUDE'S experiments the values of h/λ lie between ·065 and ·332.

In LOBACH'S experiments the values of h/λ lie between ·042 and ·167.

Hence, in the two sets of experiments, the greatest value of the modulus of $e^{-2\iota \mathrm{M}h}$ is about ·1838, and therefore the greatest value of the modulus of $\left(\frac{1-x}{1+x}\right)^2 e^{-2\iota \mathrm{M}h}$ is about ·1035 ; this corresponds to the thinnest film, for the thicker films the modulus is very much smaller.

Hence if we neglect the fourth term in the numerator and the second term in the denominator, and put

$$\mathrm{F}/\mathrm{E} = \tfrac{1}{2}\mathrm{H}'\eta_3 \Big[\{(1-x)/(1+x)\}\, \{1 - e^{-2\iota \mathrm{M}h}\} + \iota\,.\, \mathrm{M}h\Big]$$

we have an approximate formula whose error, for the very thinnest film considered, will not exceed about 10 per cent., and is very much smaller for the large majority of the experiments.

Putting in numerical values, this becomes (for iron, and sodium light),

$$\mathrm{F}/\mathrm{E} = -\iota\,.\, c\lambda \mathrm{C}_0 \gamma_0 e^{\iota x} \mathrm{R}^{-2} e^{-2\iota\alpha} \Bigg[\Big\{(3\cdot 21) + \iota (2\cdot 36)\Big\} 2\pi h/\lambda$$
$$+ (\cdot 7500)\, e^{-\iota (23^\circ 21')} \Big\{1 - e^{-\{(3\cdot 21) + \iota (2\cdot 36)\} 4\pi h/\lambda}\Big\}\Bigg].$$

In the experiments the film is generally magnetised as strongly as possible, but there is no direct way of ascertaining the intensity of magnetisation attained. Thus γ_0 is to a certain extent indeterminate. According to EWING the maximum intensity of magnetisation for some specimens of iron is about 1730 C.G.S. units. I shall therefore assume $\gamma_0 = 1730$; also I take C_0 as determined by $\log_{10}(-\mathrm{C}_0) = \overline{11}\cdot 8623$.

In one of LOBACH'S experiments the thickness of the film is given by

$$h = 82 \times 10^{-7} \text{ centim.}$$

The light used is sodium light, so that $\lambda = 5896 \times 10^{-8}$ centim. ; and the rotation (θ) in the direction of the magnetising current is observed to be 1·62 degrees.

To compare this with the rotation indicated by theory, we notice that

$$h/\lambda = \cdot 1391, \quad \text{and} \quad 2\pi h/\lambda = \cdot 8740.$$

So that

$$F/E = -\iota . c\lambda C_0\gamma_0 e^{\iota x} R^{-2} e^{-2\iota\alpha}\left[(2\cdot 805) + \iota(2\cdot 063) + (\cdot 7500)\, e^{-\iota(23^\circ 21')}\right]$$

(the other term being in this case so small that it may be neglected)

$$= -\iota . c\lambda C_0\gamma_0 e^{\iota x} R^{-2} e^{-2\iota\alpha}(3\cdot 934)\, e^{\iota(27^\circ 21')}$$

$$= c\lambda C_0\gamma_0 \frac{3\cdot 934}{15\cdot 87}\, e^{\iota\{x-2\alpha-90^\circ+27^\circ 21'\}}$$

$$= c\lambda C_0\gamma_0 \frac{3\cdot 934}{15\cdot 87}\, e^{\iota(107^\circ 41')}$$

x being taken as 63°.

The angle θ is the real part of this complex, and is therefore ·01677 in circular measure, being positive when γ_0 is positive, so that it is a rotation in the direction of the magnetising current. This theoretical value of θ in degrees is ·961. As we have seen, the observed value in degrees is 1·62 ; in comparing these results it should be noticed that one factor of the theoretical value of θ is the cosine of an angle which is just about 17° greater than a right angle ; this angle contains x, whose value we have had to guess ; a comparatively small error in the value assigned to x will therefore make a considerable error in the calculated value of θ. The values of C_0 and γ_0 being also uncertain, the agreement of the theory with experiment may be regarded as good.

In one of DRUDE's experiments $h/\lambda = \cdot 332$, and the light used is red ; the observed rotation is 4·25 degrees. If we substitute this value of h/λ in the above-obtained approximate formula, we find

$$F/E = -\iota . c\lambda C_0\gamma_0\, e^{\iota x} R^{-2} e^{-2\iota\alpha}(8\cdot 715)\, e^{\iota(32^\circ 3')}$$

$$= c\lambda C_0\gamma_0 \frac{8\cdot 715}{15\cdot 87}\, e^{\iota(112^\circ 23')}$$

whence $\theta = \cdot 05187$ in circular measure, or 2·972 degrees.

In this case, in addition to the possible causes of error referred to in connexion with the previous experiment, it is to be noticed that though the experiment was made with red light, it has been necessary in the calculation to use the values of C_0, x, R, and α for yellow light, for lack of information as to their values in the case of red light. When also it is borne in mind that the value of the magneto-optic constant derived from observation of reflection from mirrors has here been applied to test experiments on transmission through thin films, with results not only of the same order of magnitude but identical within the limits of uncertainty of the

intensity of magnetisation, the agreement must be considered as a very satisfactory vindication both of the theory and of the experiments.

Conclusion.

31. The various results obtained in this paper do not, I think, require any detailed comment. They may be fairly claimed to shew a remarkably good agreement between theory and experiment, a better agreement, I believe, than is shewn in the papers of GOLDHAMMER and DRUDE. The only considerable discrepancy arises in connexion with the original KERR experiments; but here it is to be remembered that the experiments of KERR, and those of SISSINGH and ZEEMAN, are not measurements of different phenomena, but are different ways of measuring the same phenomenon. Hence any theory that agrees with one of these sorts of experiments ought to agree equally well with the other sort; and if this is found not to be the case it is probably not the fault of the theory, but must be attributed to inaccuracy in one of the sets of experimental results. Thus it would appear that the original experiments of KERR, who was the pioneer in this subject, have been quantitatively much improved on by later investigations.